Hierarchical four-body dynamics in post-Newtonian regime

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Motion in Keplerian potential

Shape of the orbit:

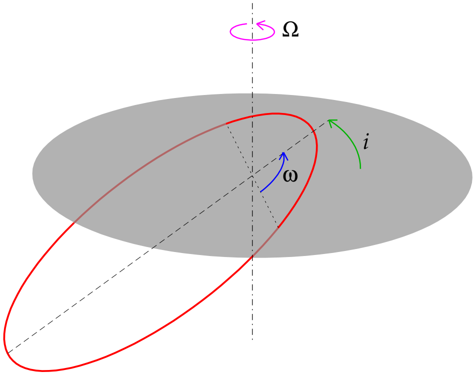
- semi-major axis, a
- eccentricity, e

Orientation of the orbital plane:

- inclination, $i \in (0°, 180°)$
- longitude of the ascending node, $\Omega \in (0°, 360°)$

Orientation and position within the orbital plane:

- argument of pericentre, ω
- true anomaly, ν

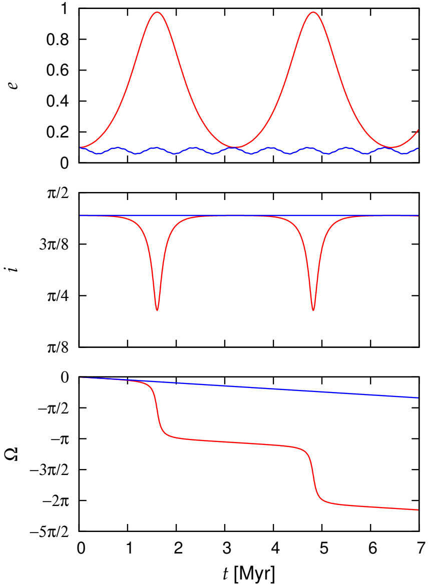


Kozai-Lidov oscillations

Broken spherical symmetry $\rightarrow$ angular momentum is not an integral of motion $\rightarrow$ oscillations of eccentricity and inclination

Example:

- $M_{\rm BH} = 3.5 \times 10^6 M_\odot$
- $M_{\rm CND} = M_{\rm BH}$, $R_{\rm CND} = 1.5\,\mathrm{pc}$
- $a_0 = 0.1 R_{\rm CND}$, $e_0 = 0.1$ $i_0 = 80^\circ$, $\omega_0 = 0$, $\Omega_0 = 0$
- $M_{\rm c} = 0.1 M_{\rm BH}$



'Kozai' equations

$$T_{\mathrm{K}}\sqrt{1-e^2}\;\frac{\mathrm{d}e}{\mathrm{d}t} = \frac{15}{8}\,e\,(1-e^2)\,\sin 2\omega\,\sin^2 i$$

$$T_{\mathrm{K}}\sqrt{1-e^2}\;\frac{\mathrm{d}i}{\mathrm{d}t} = -\frac{15}{8}\,e^2\,\sin 2\omega\,\sin i\,\cos i$$

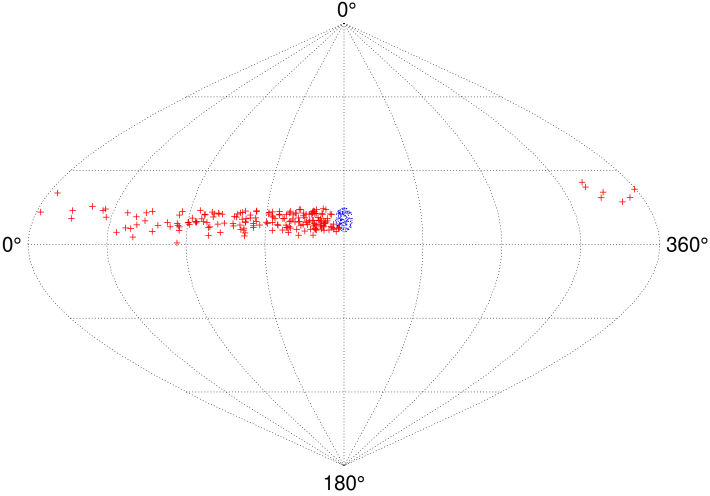
$$T_{\mathrm{K}}\sqrt{1-e^2}\;\frac{\mathrm{d}\omega}{\mathrm{d}t} = \frac{3}{4}\left\{2-2e^2+5\sin^2\omega\left[e^2-\sin^2 i\right]\right\}+\left(\frac{\mathrm{d}\omega}{\mathrm{d}t}\right)_{\mathrm{c}}$$

$$T_{\mathrm{K}}\sqrt{1-e^2}\;\frac{\mathrm{d}\Omega}{\mathrm{d}t} = -\frac{3}{4}\cos i\left[1+4e^2-5e^2\cos^2\omega\right]$$

$$T_{\mathrm{K}} \equiv \frac{M_{\mathrm{BH}}}{M_{\mathrm{CND}}}\,\frac{R^3_{\mathrm{CND}}}{a\sqrt{GM_{\mathrm{BH}}a}}$$

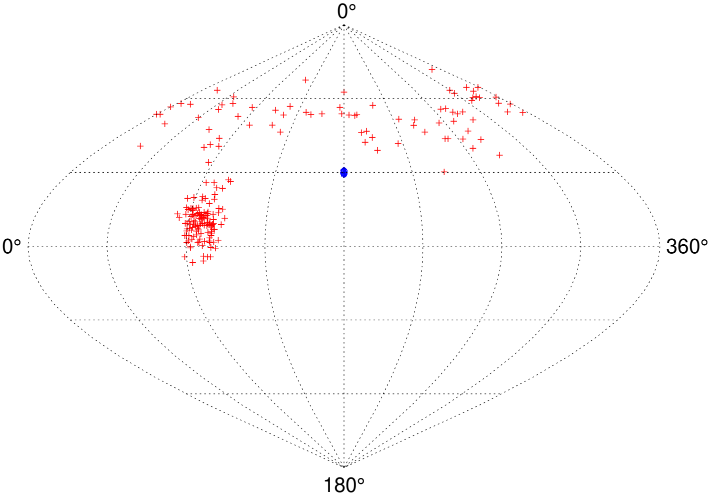
$$\frac{\mathrm{d}\Omega}{\mathrm{d}t} \approx -\frac{3}{4}\,\frac{\cos i}{T_{\mathrm{K}}}\,\frac{1+\frac{3}{2}e^2}{\sqrt{1-e^2}} \approx \textit{const} \quad \text{for} \quad \left(\frac{\mathrm{d}\omega}{\mathrm{d}t}\right)_{\mathrm{c}} \quad \text{sufficiently large}$$

Warped disc of test particles



(Šubr, Schovancová & Kroupa 2009)

Warped self-gravitating disc



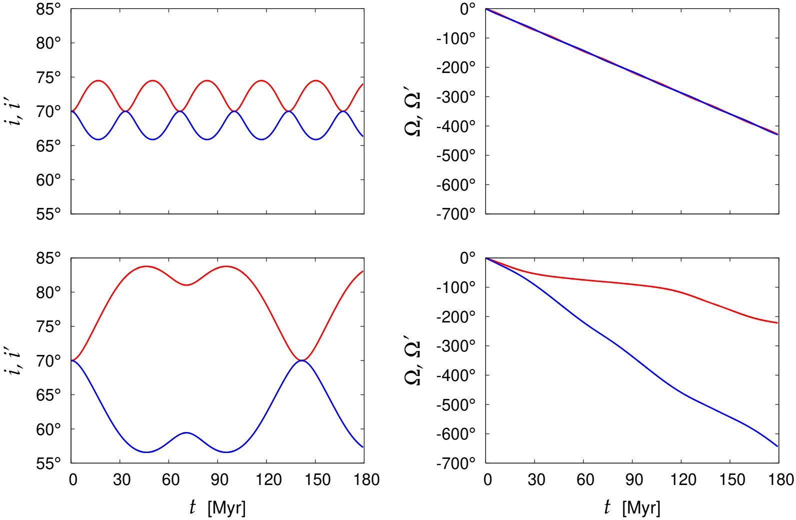
(Haas, Šubr & Kroupa 2011)

Two interacting stars in a perturbed Keplerian potential

Secular theory for evolution of two nearby orbits in a perturbed Keplerian potential developed in
Haas, Šubr & Vokrouhlický (2011).

- dominating Keplerian potential ($M_{\mathrm{BH}} = 3.5 \times 10^6 M_\odot$)
- perturbing body on circular orbit ($M_{\mathrm{CND}} = 0.3\, M_{\mathrm{BH}}$, $R_{\mathrm{CND}} = 1.5\,\mathrm{pc}$)
- extended spherical potential damping the Kozai-Lidov oscillations
- two light bodies on initially nearby orbits
  - $i = i' = 70^\circ$
  - $\Omega = \Omega' = 0$
  - $e = e' = 0$
  - $a = 0.05\, R_{\mathrm{CND}}$, $a' = 0.04\, R_{\mathrm{CND}}$
  - $m = m' = 9 \times 10^{-6} M_{\mathrm{BH}} \ \vee \ 5 \times 10^{-6} M_{\mathrm{BH}}$

Two interacting stars in a perturbed Keplerian potential

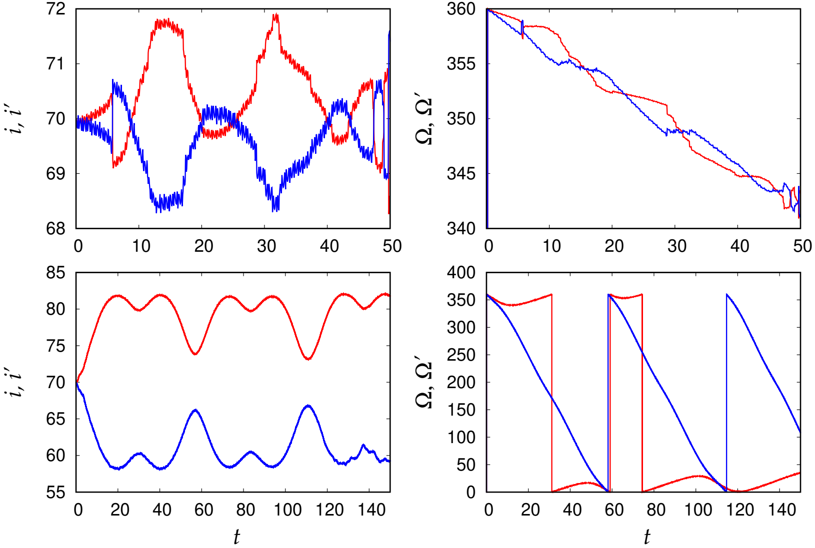


(Haas, Šubr & Vokrouhlický 2011)

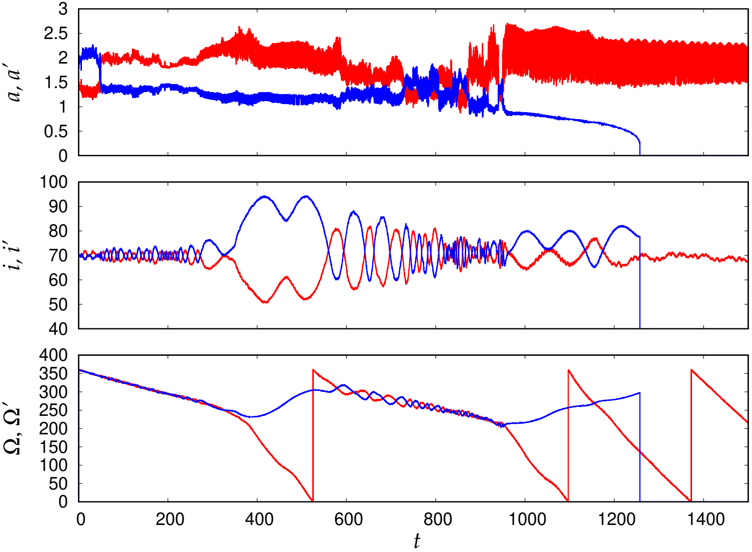
Two stars in perturbed post-Newtonian potential

Direct integrations of equations of motion by means of ARCHAIN code (Chassonnery, Capuzzo-Dolcetta & Mikkola, 2019; algorithm by Mikkola & Merritt, 2006, 2008), involving post-Newtonian terms up to the 2.5 order.

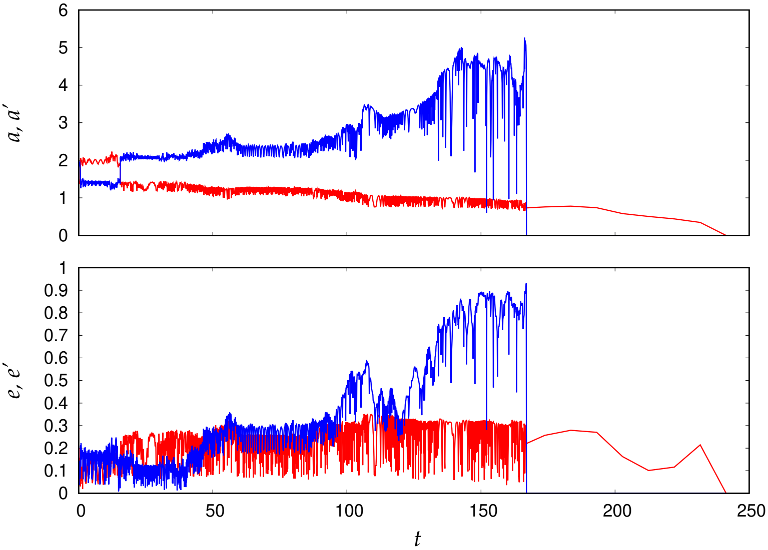
Two stars in perturbed post-Newtonian potential



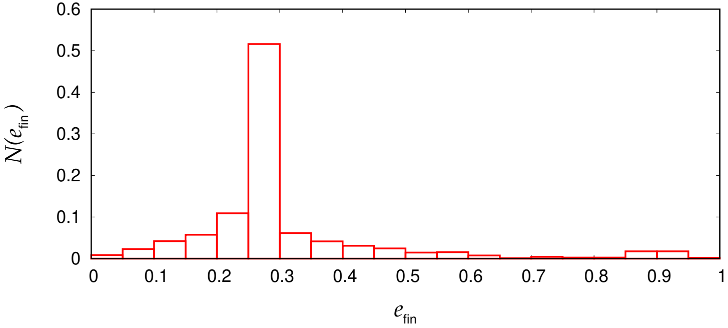
Swap, re-binding, inspiral



High eccentricity inspiral?



High eccentricity inspiral?



Summary

- hierarchical four-body dynamics in P-N exhibits similar modes of evolution to the Newtonian case (with arbitrary damper of Kozai-Lidov oscillations)
- mutual interaction of ‘light’ bodies often accelerates inspiral
    - energy transfer?
    - eccentricity oscillations?
    - effectively larger mass?
- inspiral with large (> 0.8) eccentricity in non-negligible number ($\approx 4\%$) of cases
- results very preliminary with arbitrary parameters of the system
    - revision needed
    - seek for astrophysically relevant application

High eccentricity inspiral?