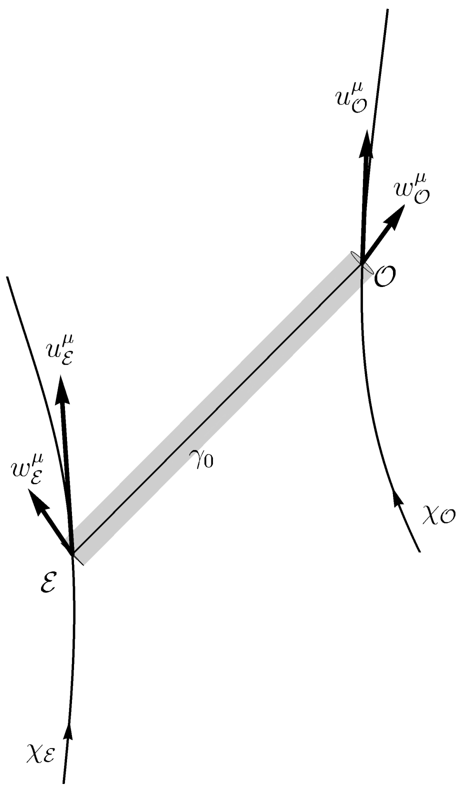
Position drift and redshift drift in general relativity



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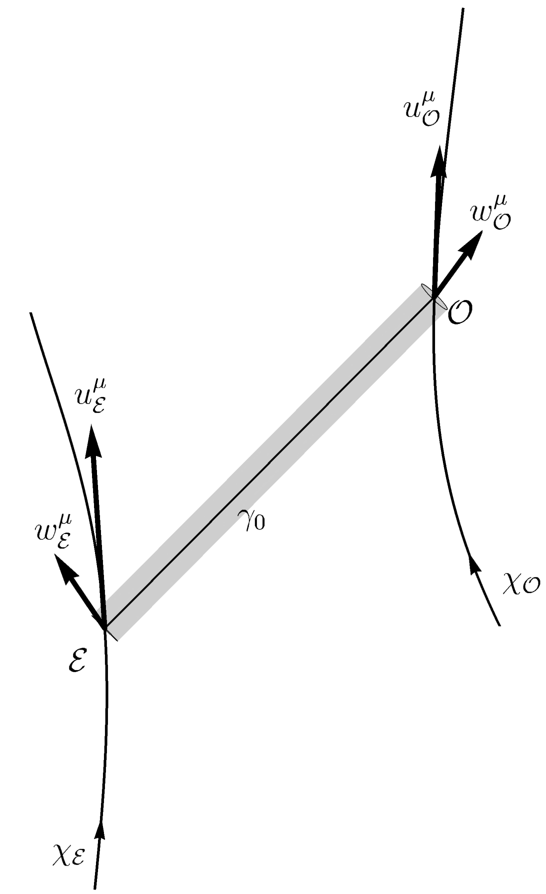
Cologne-Prague-Brno Meeting, June 1st-3rd 2022
Institute of Theoretical Physics and Astrophysics, Faculty of Science, Masaryk University, Brno

Idea



Light propagation effect in curved spacetimes (geometric optics approximation) done exactly

Focus on drift effects: temporal variations of observed position (proper motion) and redshift (redshift drift), for a given source and observer

Exact expressions, valid in any spacetime: all retardation, light bending and lensing effects taken into account automatically (good starting point for other approximations)

Geometric formulation: expressions in terms of the curvature tensor along the line of sight, plus kinematical variables (compare the Sachs optical equations [Sachs 1961])

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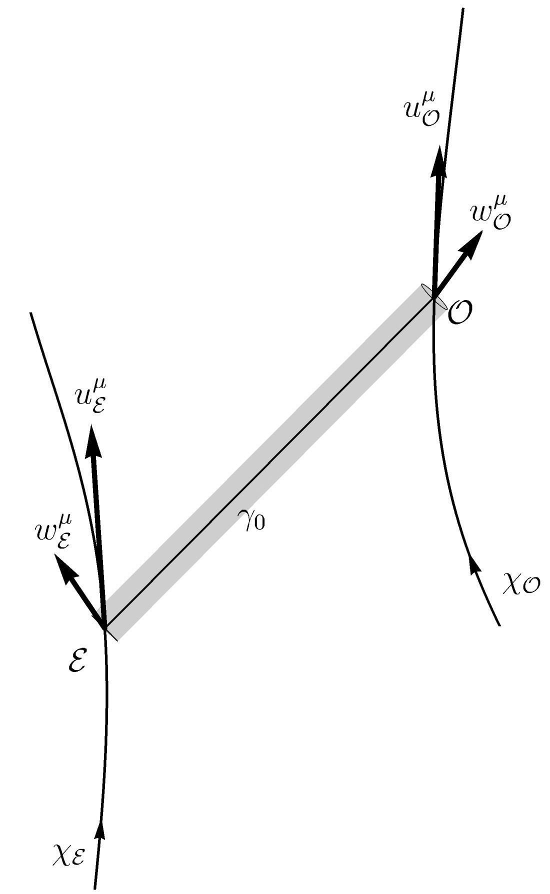


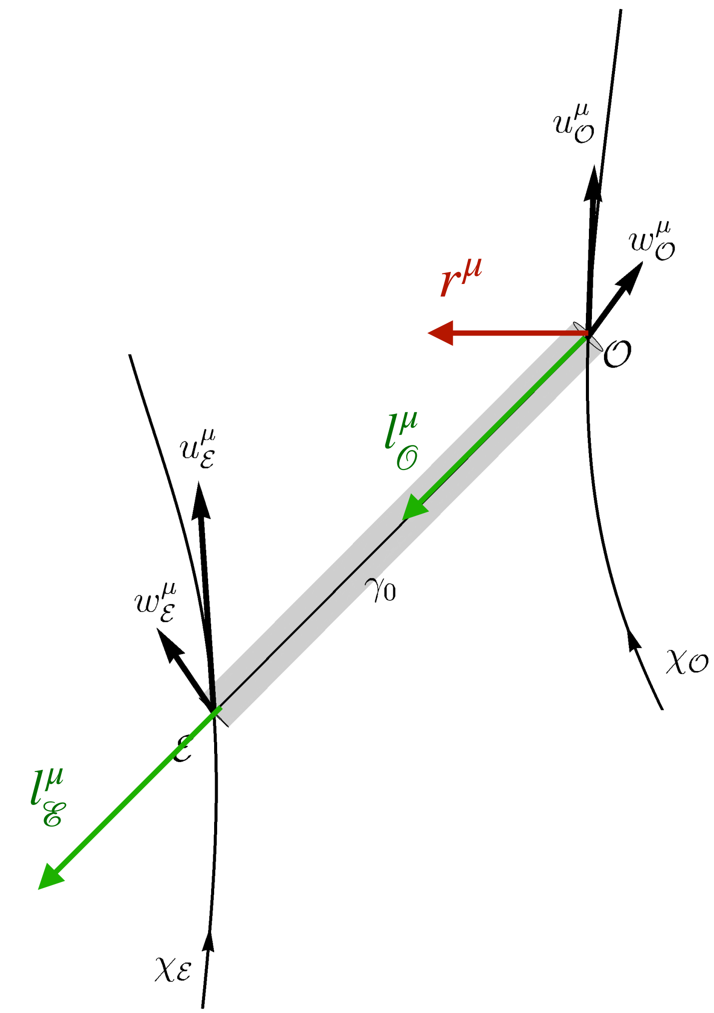
Applications:

- redshift drift and cosmic parallax in cosmology
- ray-tracing near a black hole, images of moving sources around a black hole
- pulsar timing, gravitational waves
- astrometry

Position drift

Momentary position on the sky = spatial normalized vector

$$r^\mu = \frac{l^\mu_{\mathcal{O}}}{l_{\mathcal{O}\,\nu}\, u^\nu_{\mathcal{O}}} + u^\mu_{\mathcal{O}} \qquad r^\mu\, u_{\mathcal{O}\,\mu} = 0 \qquad r^\mu\, r_\mu = 1$$



In order to define the position drift we need „fixed direction on the sky”

Many choices possible

Reasonable assumption: conserved angle between fixed directions

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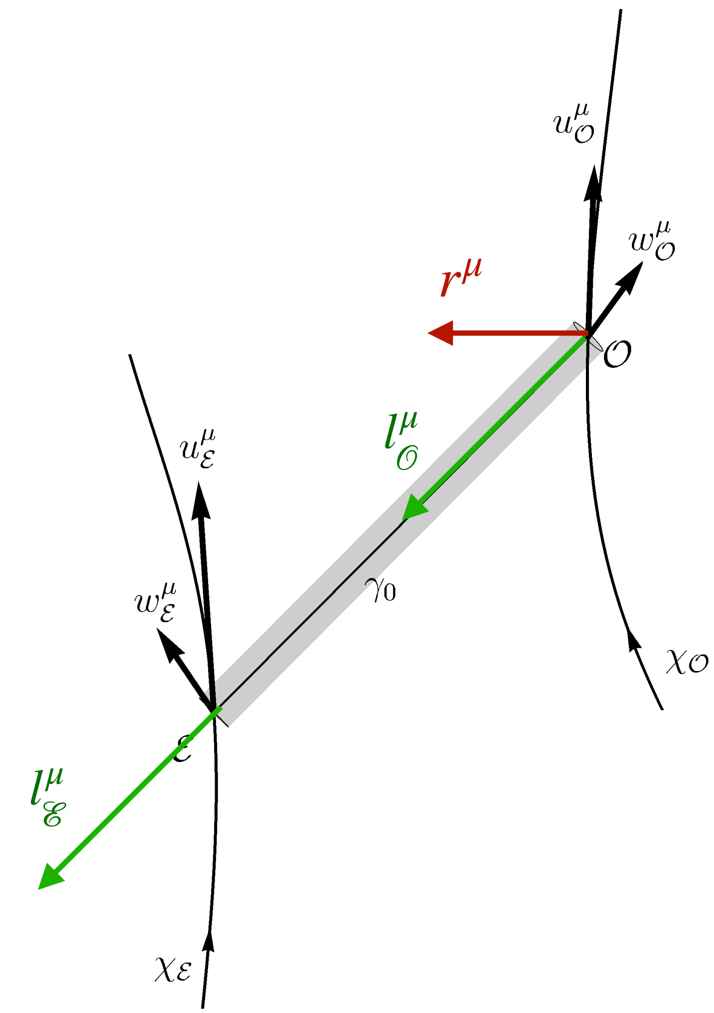

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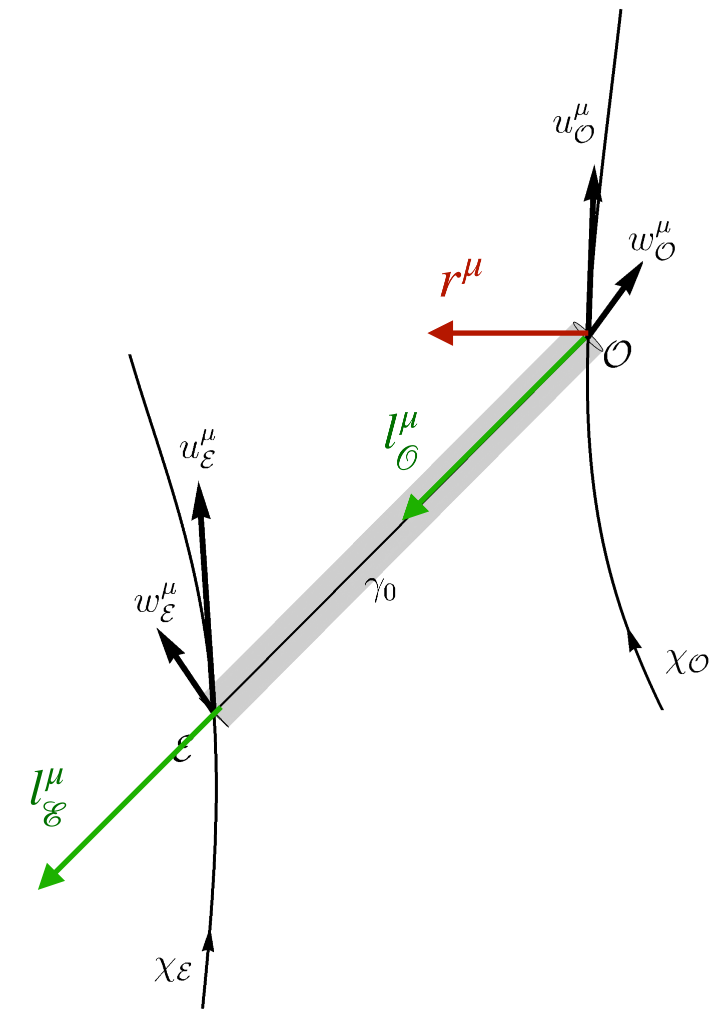

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    Gravity Probe B

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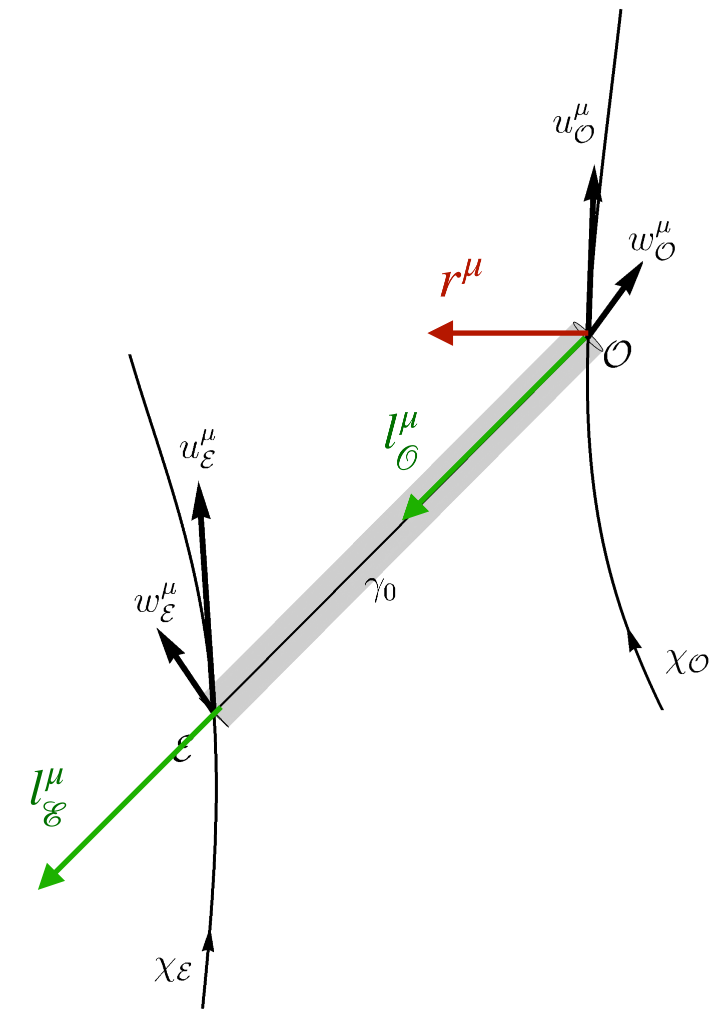

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Alternatives:

- Use local physics near the observer to define them (for example: gyroscopes)

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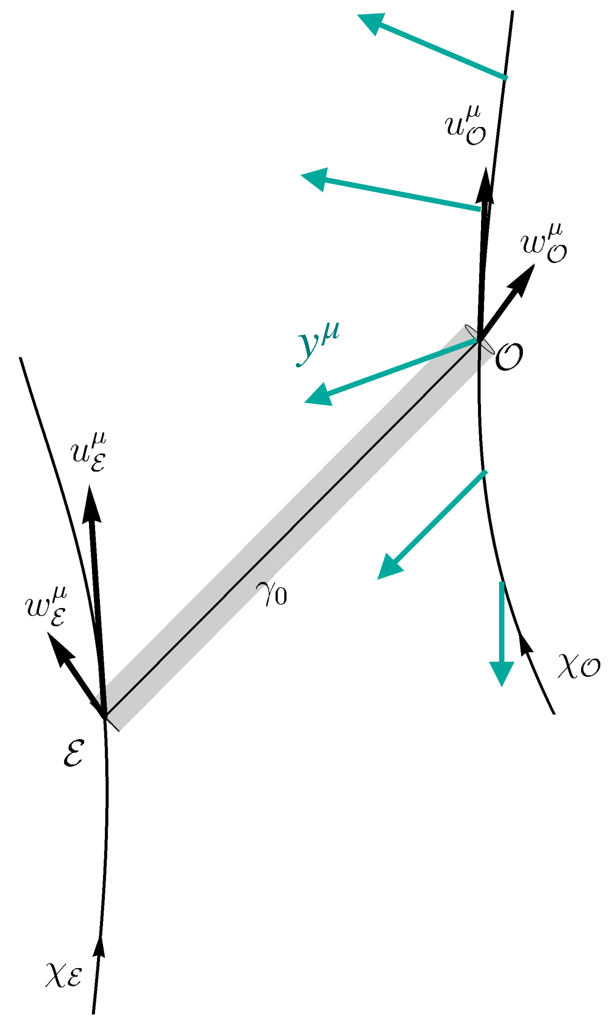
- Use positions of distant objects („fixed quasars”)

  Standard method in astrometry (International Celestial Reference Frame, Gaia Celestial Referece Frame)…

Position drift



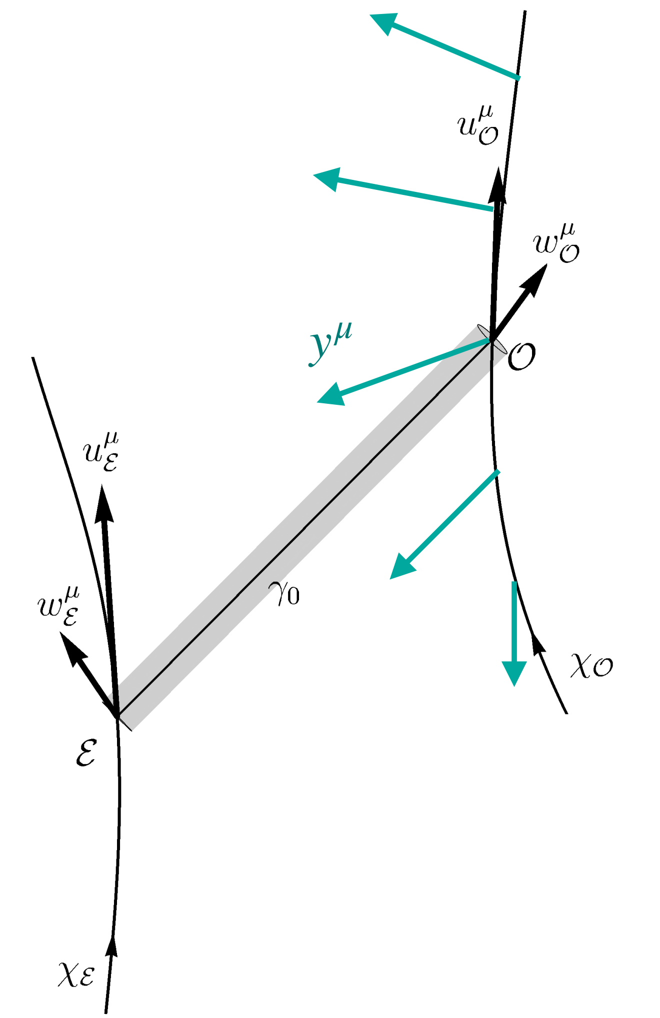
Position drift

We choose the Fermi-Walker transport and derivative (local physics/geometry)

[Hellaby, Walters 2018]

Fermi-Walker derivative: $\delta_{\mathcal{O}} y^\mu = \nabla_{u_{\mathcal{O}}} y^\mu - u^\mu_{\mathcal{O}} \, w_{\mathcal{O}\,\nu} \, y^\nu$

For geodesic observers it agrees with the covariant derivative



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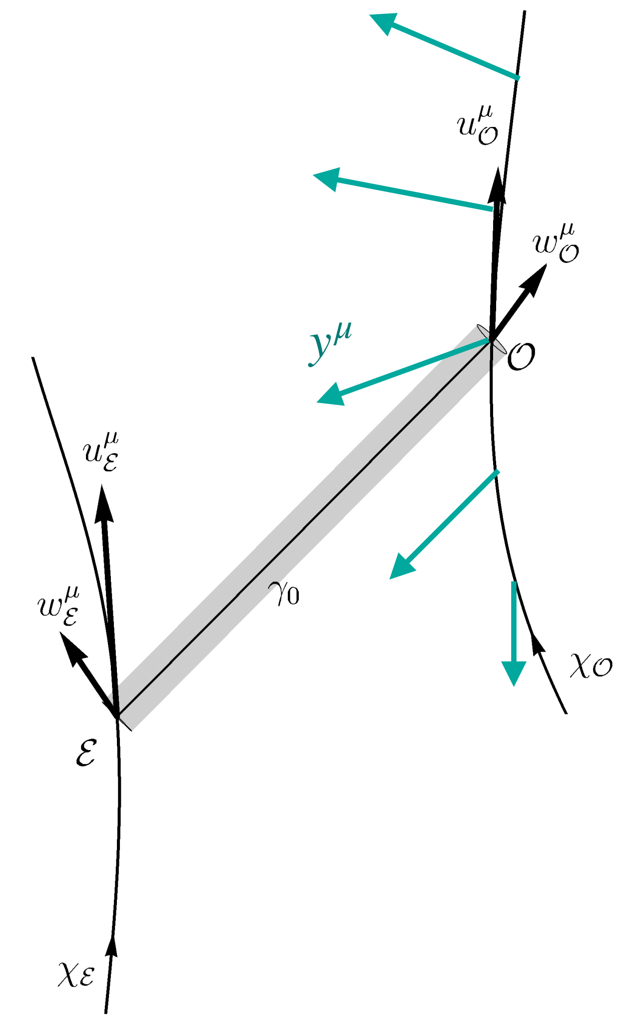
$$\delta_{\mathcal{O}}\, g_{\mu\nu} = 0$$

$$y^\mu\, u_{\mathcal{O}\,\mu} = 0 \Longrightarrow \delta_{\mathcal{O}} y^\mu\, u_{\mathcal{O}\,\mu} = 0$$

Angles on the celestial sphere conserved



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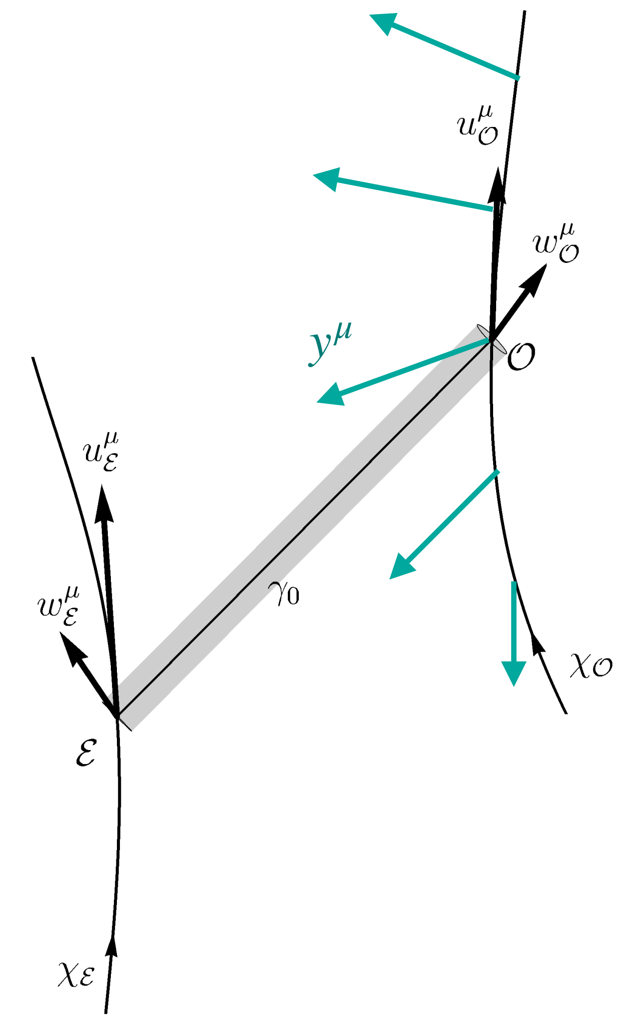

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Angles on the celestial sphere conserved

Other reasonable definitions differ by a rotation

$$\tilde{\delta} r^i = \delta_{\mathcal{O}} r^j + \Omega^i{}_j\, r^j \qquad\qquad \Omega_{ij} = -\,\Omega_{ji}$$

Physical situation

worldlines of the observer and emitter

$$\chi_{\mathscr{O}}(\tau)$$

$$\chi_{\mathscr{E}}(\tau')$$

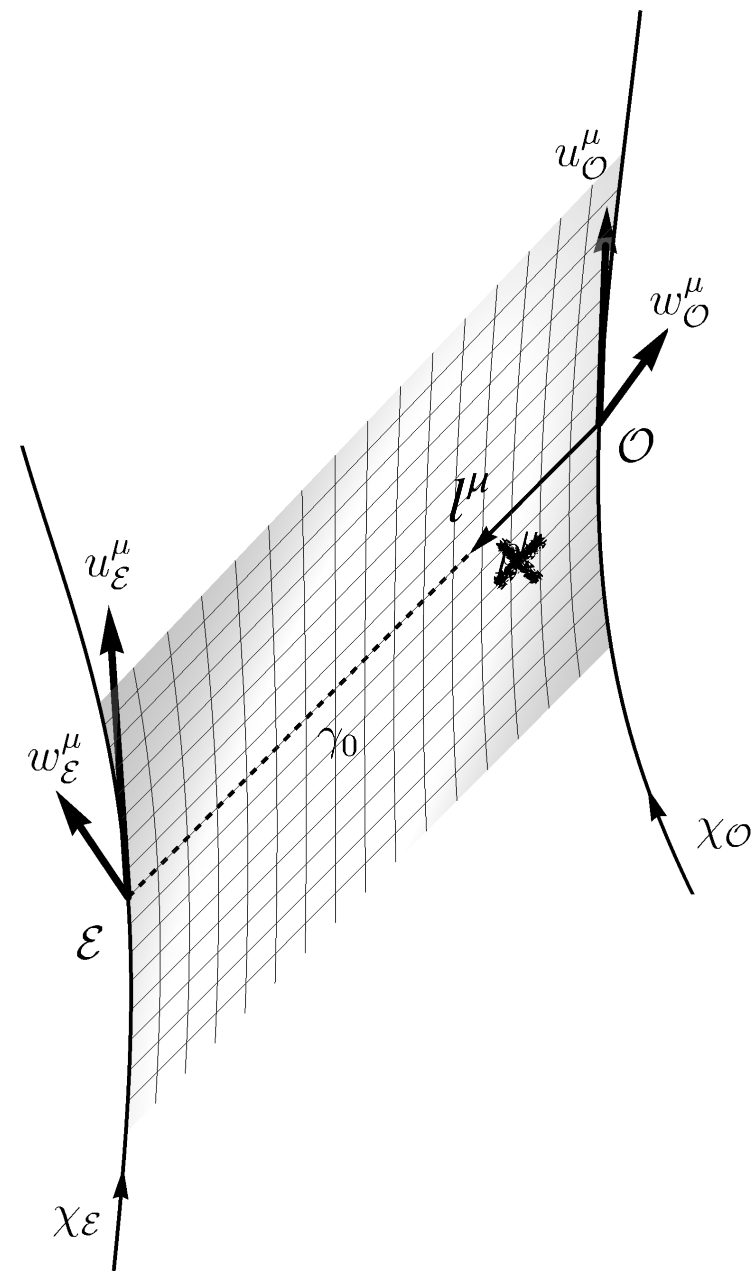
connecting null geodesics

$$\gamma_\tau(\lambda)$$

affine parametrization

$$\gamma_\tau(\lambda_{\mathscr{O}}) = \chi_{\mathscr{O}}(\tau)$$

$$\gamma_\tau(\lambda_{\mathscr{E}}) = \chi_{\mathscr{E}}(s(\tau))$$



tangent vectors to null geodesics $\quad l^\mu = \dfrac{\partial x^\mu}{\partial \lambda}$

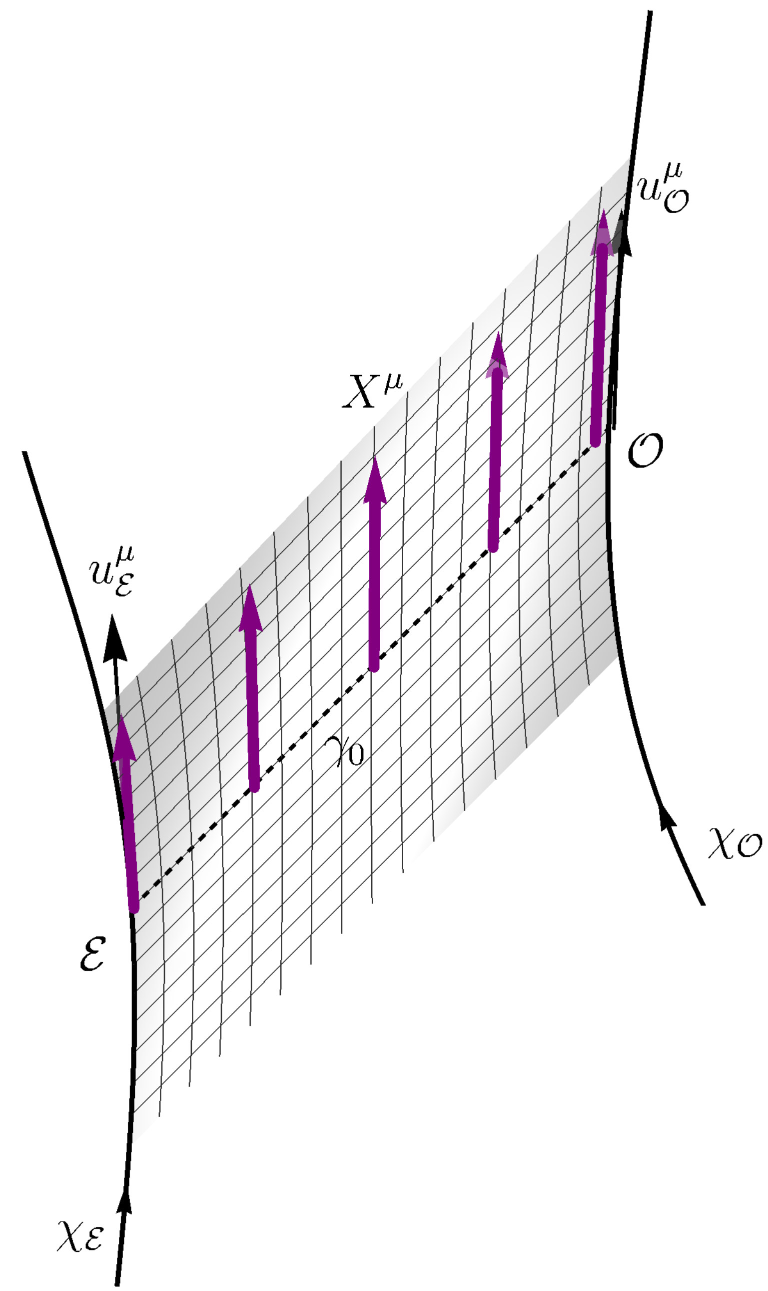
observation time vector $\quad X^\mu = \dfrac{\partial x^\mu}{\partial \tau}$

find an expression (up to $C\, l^\mu$)
in terms of $u_{\mathscr{E}}$, $u_{\mathscr{O}}$ and functionals of curvature

Observation time vector

X^μ satisfies the GDE

$$\nabla_l \nabla_l X^\mu - R^\mu{}_{\alpha\beta\nu}\, l^\alpha\, l^\beta\, X^\nu = 0$$



Observation time vector

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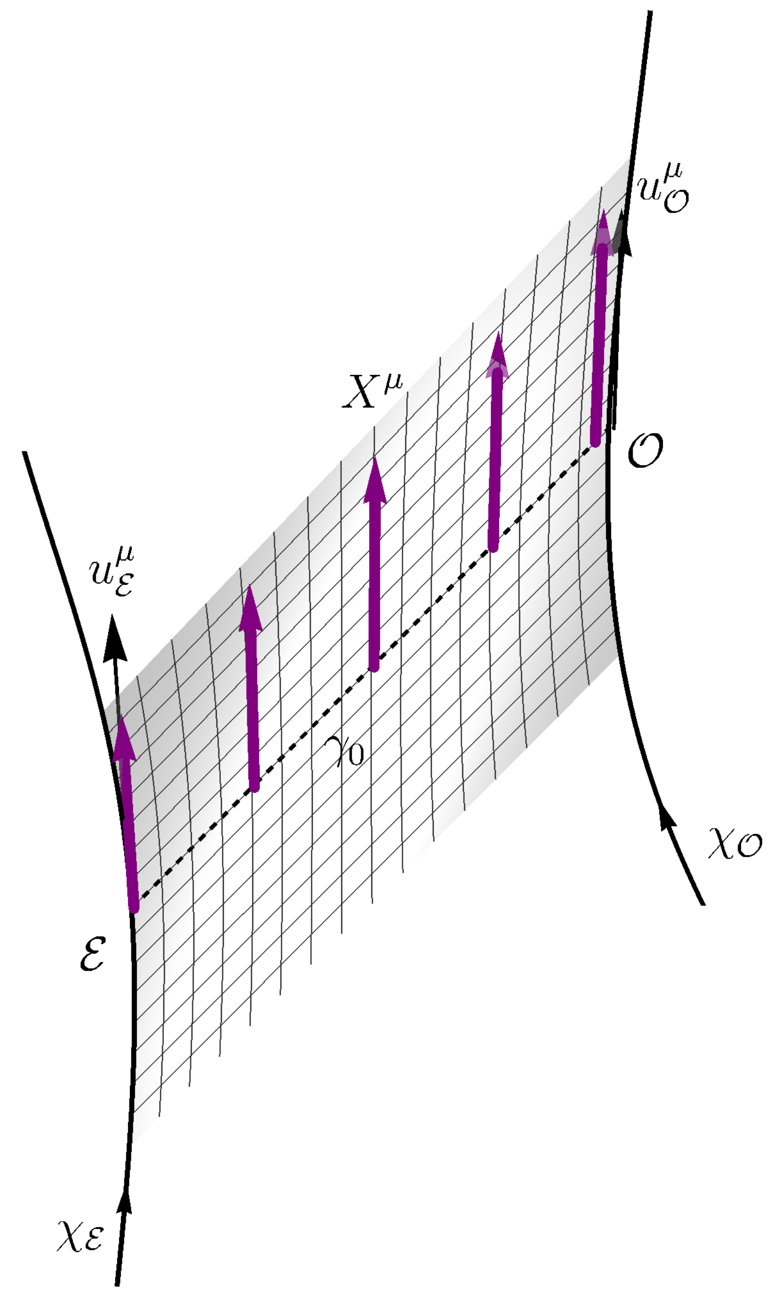
from the parametrization

$$X^\mu(\mathscr{O}) = u^\mu_{\mathscr{O}}$$

$$X^\mu(\mathscr{E}) = C\, u^\mu_{\mathscr{E}}$$

$$\nabla_X l^\mu\, l_\mu = 0 \qquad \Longleftrightarrow l_{\mathscr{O}\,\mu}\, X^\mu(\mathscr{O}) = l_{\mathscr{E}\,\mu}\, X^\mu(\mathscr{E})$$

$$\frac{ds(\tau)}{d\tau} = \quad C = \frac{l_{\mathscr{O}\,\mu}\, u^\mu_{\mathscr{O}}}{l_{\mathscr{E}\,\mu}\, u^\mu_{\mathscr{E}}} = \frac{1}{1+z}$$



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boundary value problem (Dirichlet)

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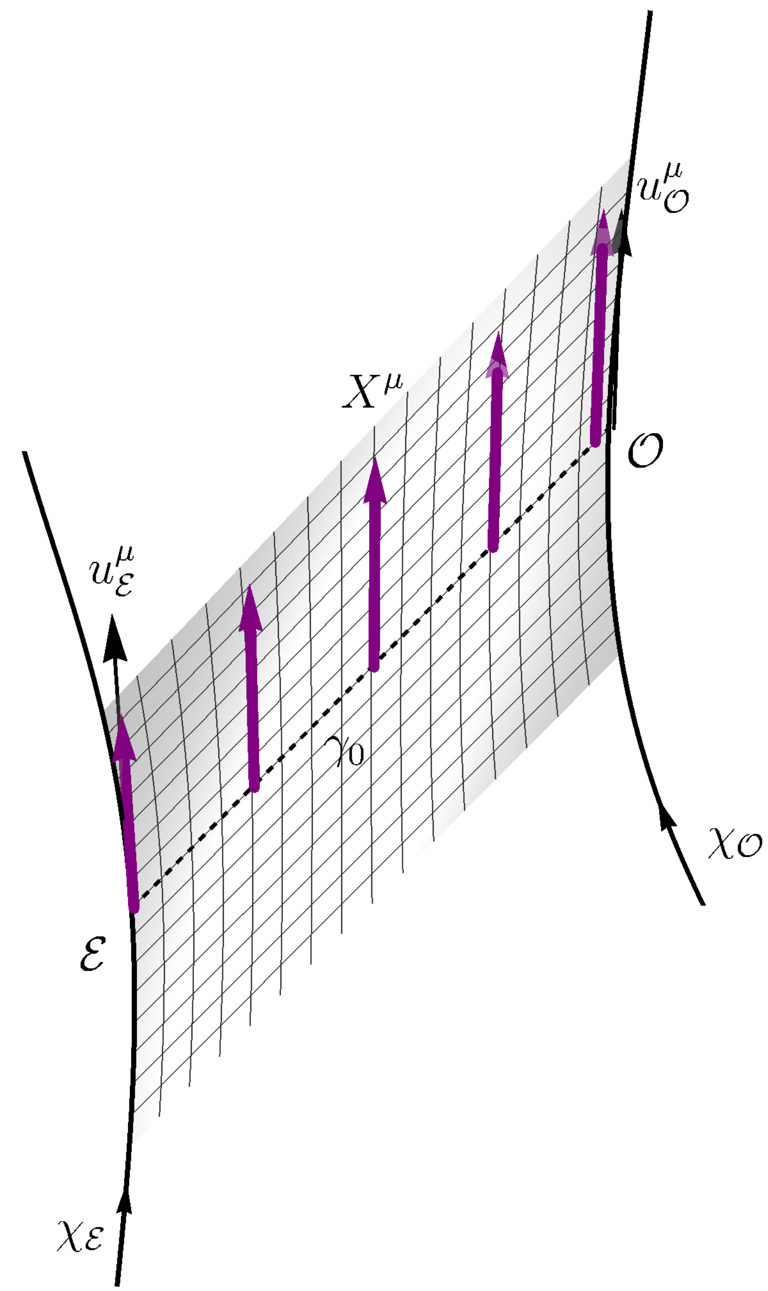


Observation time vector

Solution in terms of $u_{\mathscr{O}}$, $u_{\mathscr{E}}$ and curvature functionals

$$\ddot{\mathscr{D}}^A{}_B - R^A{}_{\alpha\beta C}\, l^\alpha\, l^\beta\, \mathscr{D}^C{}_B = 0$$

$$\mathscr{D}^A{}_B(\mathscr{O}) = 0$$

$$\dot{\mathscr{D}}^A{}_B(\mathscr{O}) = \delta^A{}_B$$

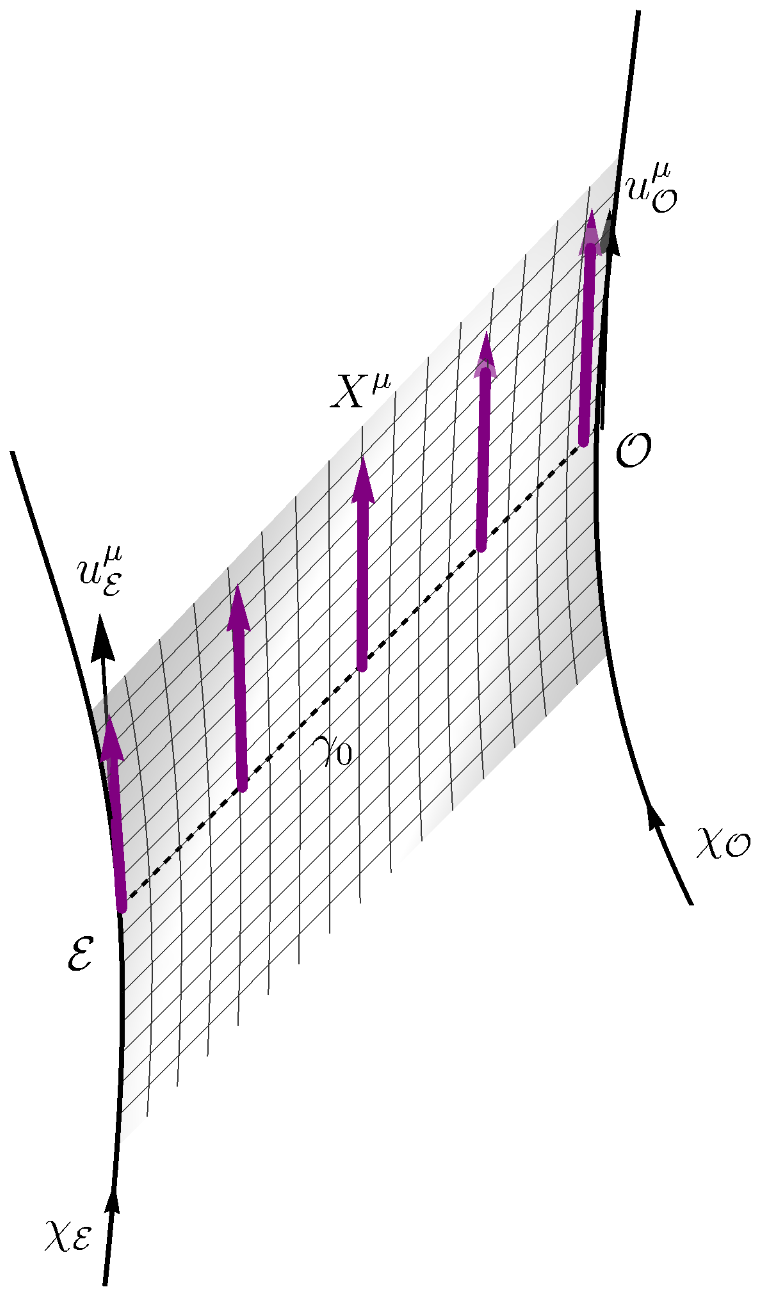
Jacobi matrix

$$\ddot{m}^A{}_\mu - R^A{}_{\alpha\beta B}\, l^\alpha\, l^\beta\, m^B{}_\mu = R^A{}_{\alpha\beta\mu}\, l^\alpha\, l^\beta$$

$$m^A{}_\mu(\mathscr{O}) = 0$$

$$\dot{m}^A{}_\mu(\mathscr{O}) = 0$$

another curvature functional vanishes in a flat space



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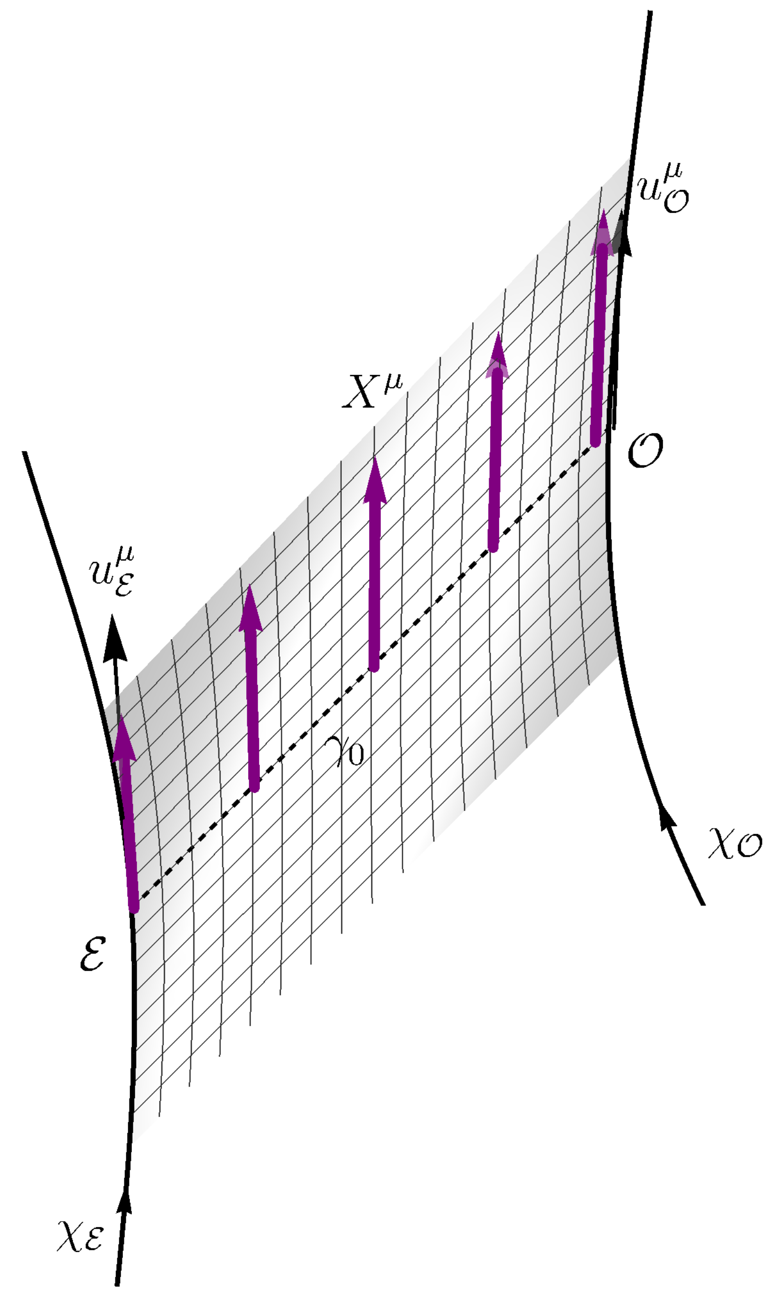


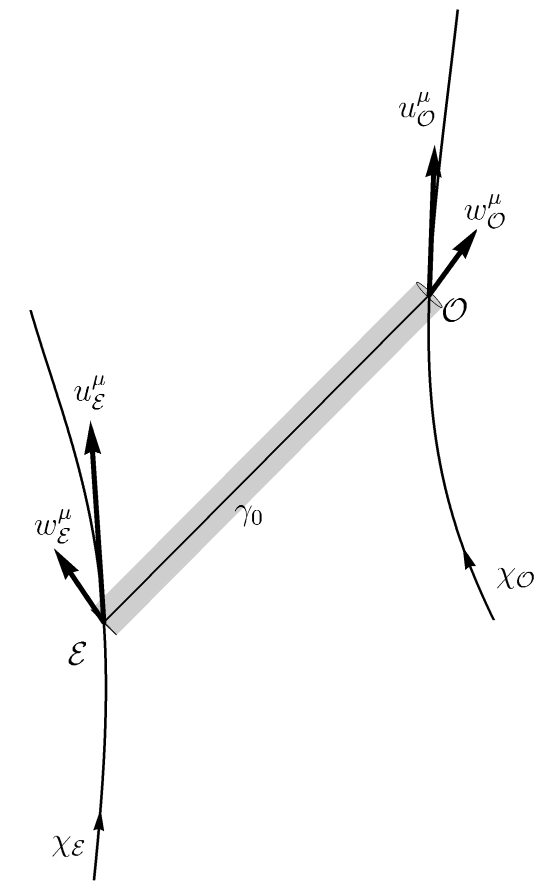
- solution

$$X^\mu = \hat{u}^\mu_{\mathscr{O}} + e^\mu_A\left(\phi^A + m^A{}_\nu\, u^\nu_{\mathscr{O}}\right) + C\cdot l^\mu$$

$$\phi^C(\lambda) = \mathscr{D}^C{}_A(\lambda)\, \mathscr{D}^{-1A}{}_B(\mathscr{E})\left(\left(\frac{1}{1+z}u_{\mathscr{E}} - \hat{u}_{\mathscr{O}}\right)^B - m^B{}_\mu(\lambda_{\mathscr{E}})\, u^\mu_{\mathscr{O}}\right)$$

Position drift formula

$\delta_{\mathcal{O}} r^{\mu} = \nabla_X r^{\mu} - u_{\mathcal{O}}^{\mu} w_{\mathcal{O}\,\nu} r^{\nu}$ Fermi-Walker derivative



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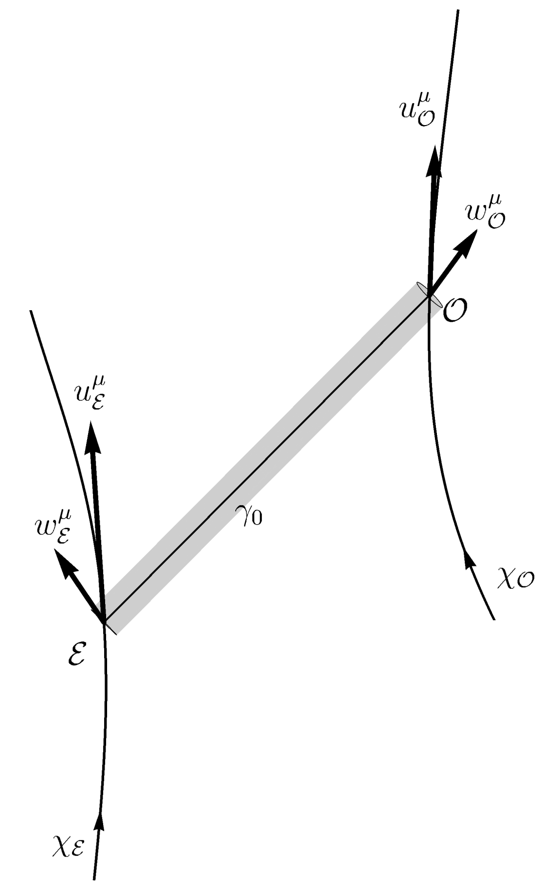

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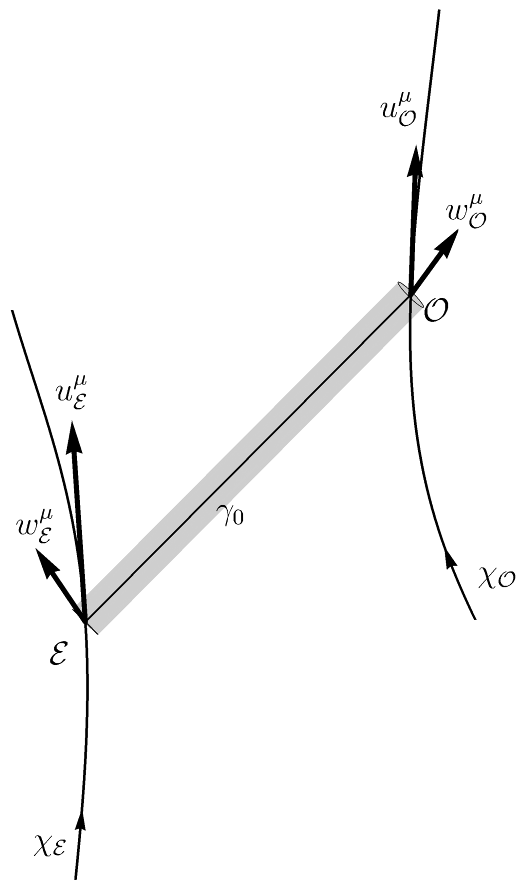
magnification matrix $M^A{}_B$



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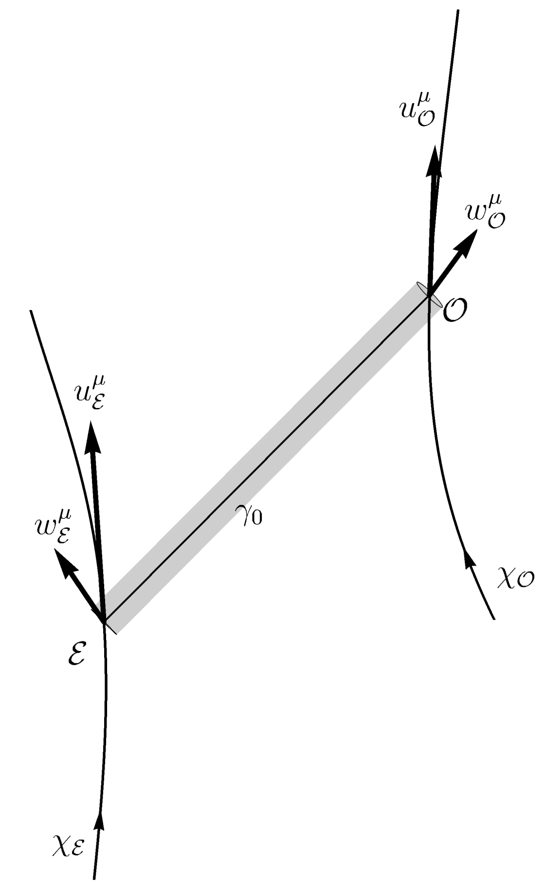
transverse 4-velocity difference



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curvature correction

$$\delta_{\mathscr{O}} r^A = (u^\mu_{\mathscr{O}}\, l_{\mathscr{O}\,\mu})^{-1}\, \mathscr{D}^{-1\,A}{}_B \left(\left(\frac{1}{1+z} u_{\mathscr{E}} - \hat{u}_{\mathscr{O}} \right)^B - m^B{}_\nu\, u^\nu_{\mathscr{O}} \right) + w^A_{\mathscr{O}}$$

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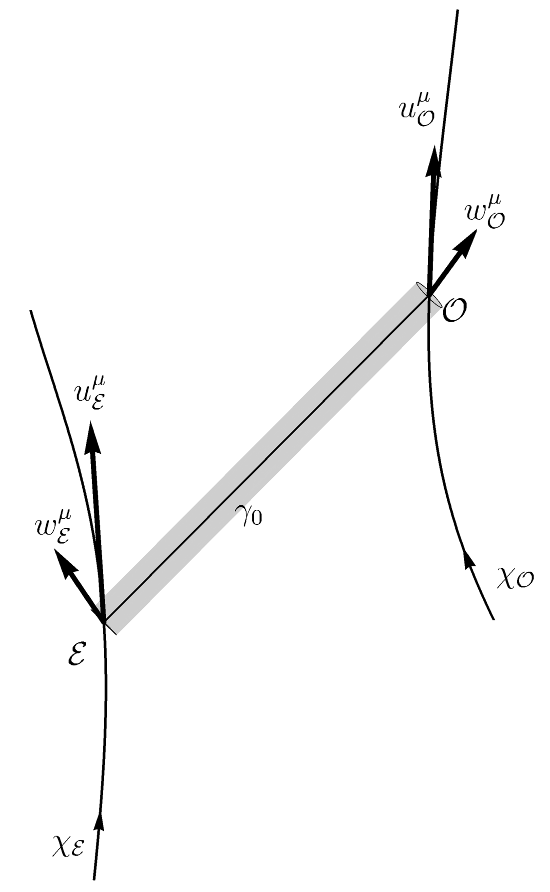

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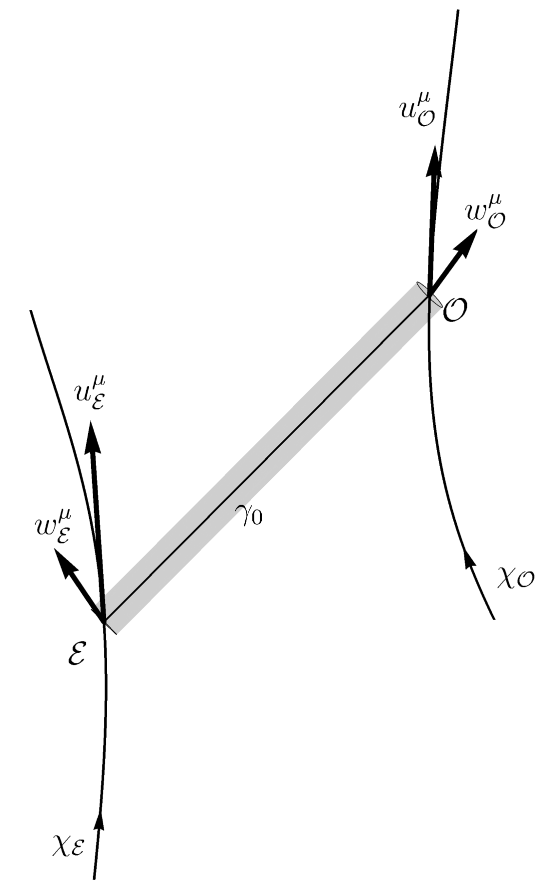
transverse 4-velocity difference

transverse components of observer's acceleration



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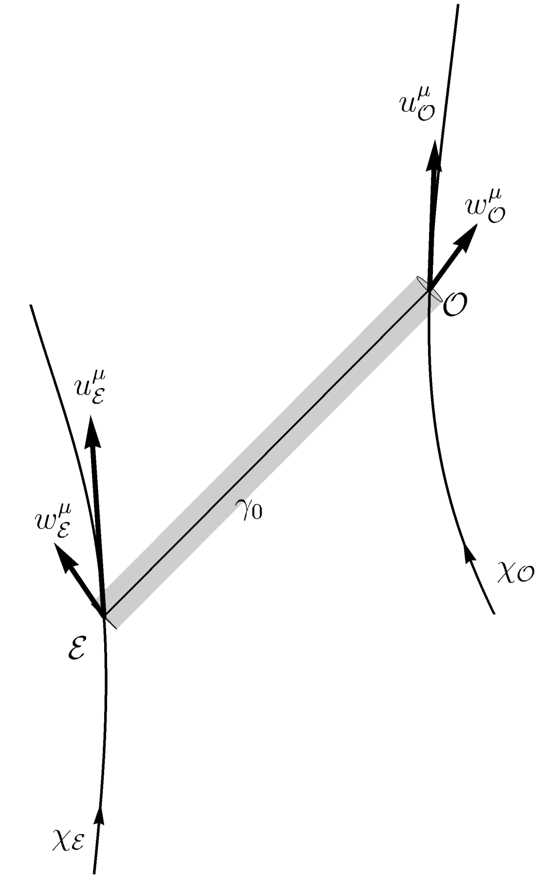
transverse 4-velocity difference

transverse components of observer's acceleration

flat spacetime

$$\delta_{\mathcal{O}} r^{A} = D^{-1}_{\mathcal{O}} \left(\frac{1}{1+z} u_{\mathcal{E}} - \hat{u}_{\mathcal{O}} \right)^{A} + w^{A}_{\mathcal{O}}$$



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transverse 4-velocity difference

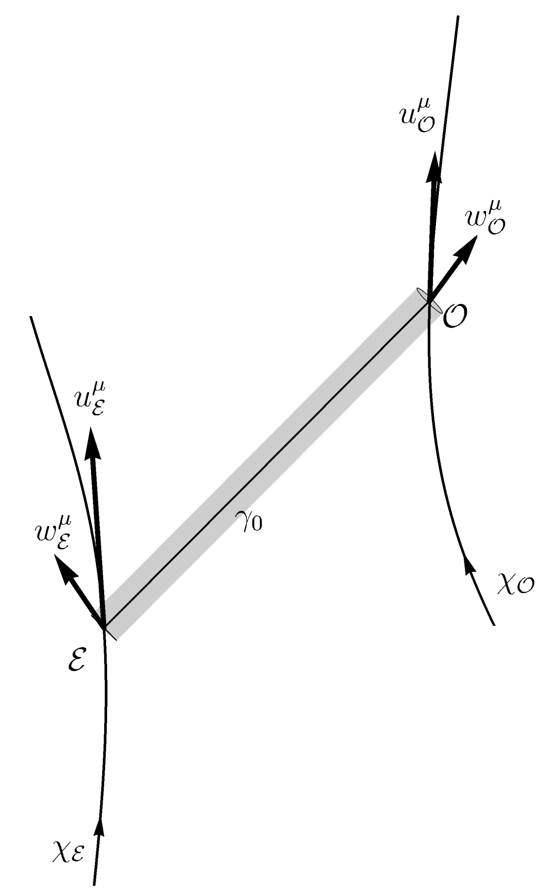
transverse components of observer's acceleration

flat spacetime

$$\delta_{\mathscr{O}} r^A = D_{\mathscr{O}}^{-1} \left(\frac{1}{1+z} u_{\mathscr{E}} - \hat{u}_{\mathscr{O}} \right)^A + w_{\mathscr{O}}^A$$

flat spacetime, non-relativistic limit $c \to \infty$

$$\delta_{\mathscr{O}} r^A = D^{-1} \, (v_{\mathscr{E}} - v_{\mathscr{O}})^A$$



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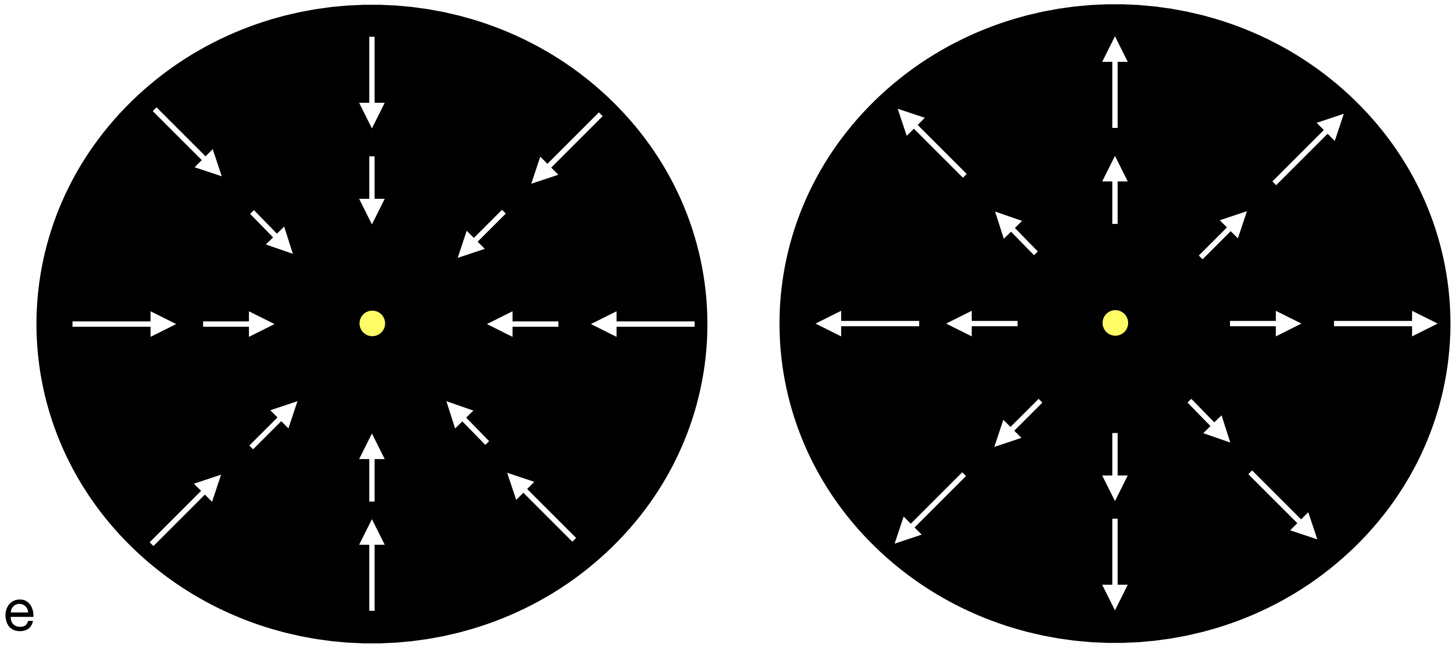
aberration drift

depends on the apparent position only

$$w^A_{\mathcal{O}} \equiv w^A_{\mathcal{O}}(r^j)$$

$$w^i_{\mathcal{O}\perp} = w^j_{\mathcal{O}}\,(\delta^i_j - r^i\, r_j)$$

vector dipole on the celestial sphere



generates conformal transformations of the celestial sphere (aberration effect)

Apparent superluminal motions

$$\delta_{\mathscr{O}} r^A = (u^\mu_{\mathscr{O}}\, l_{\mathscr{O}\,\mu})^{-1}\, \mathscr{D}^{-1\,A}{}_B \left(\left(\frac{1}{1+z} u_{\mathscr{E}} - \hat{u}_{\mathscr{O}} \right)^B - m^B{}_\nu\, u^\nu_{\mathscr{O}} \right) + w^A_{\mathscr{O}}$$

$$\delta_{\mathscr{O}} r = \gamma \beta_\perp \frac{c}{D_{\mathscr{O}}}$$

Apparent superluminal motions

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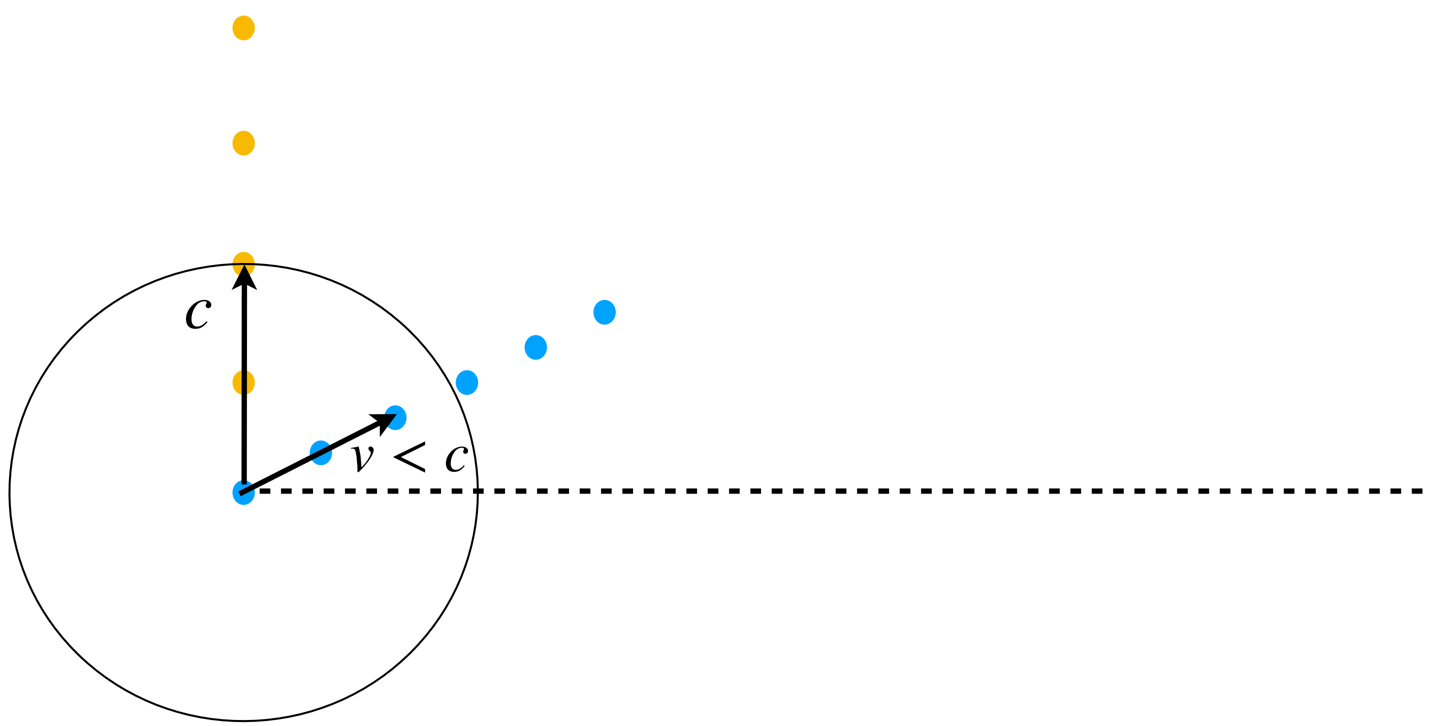
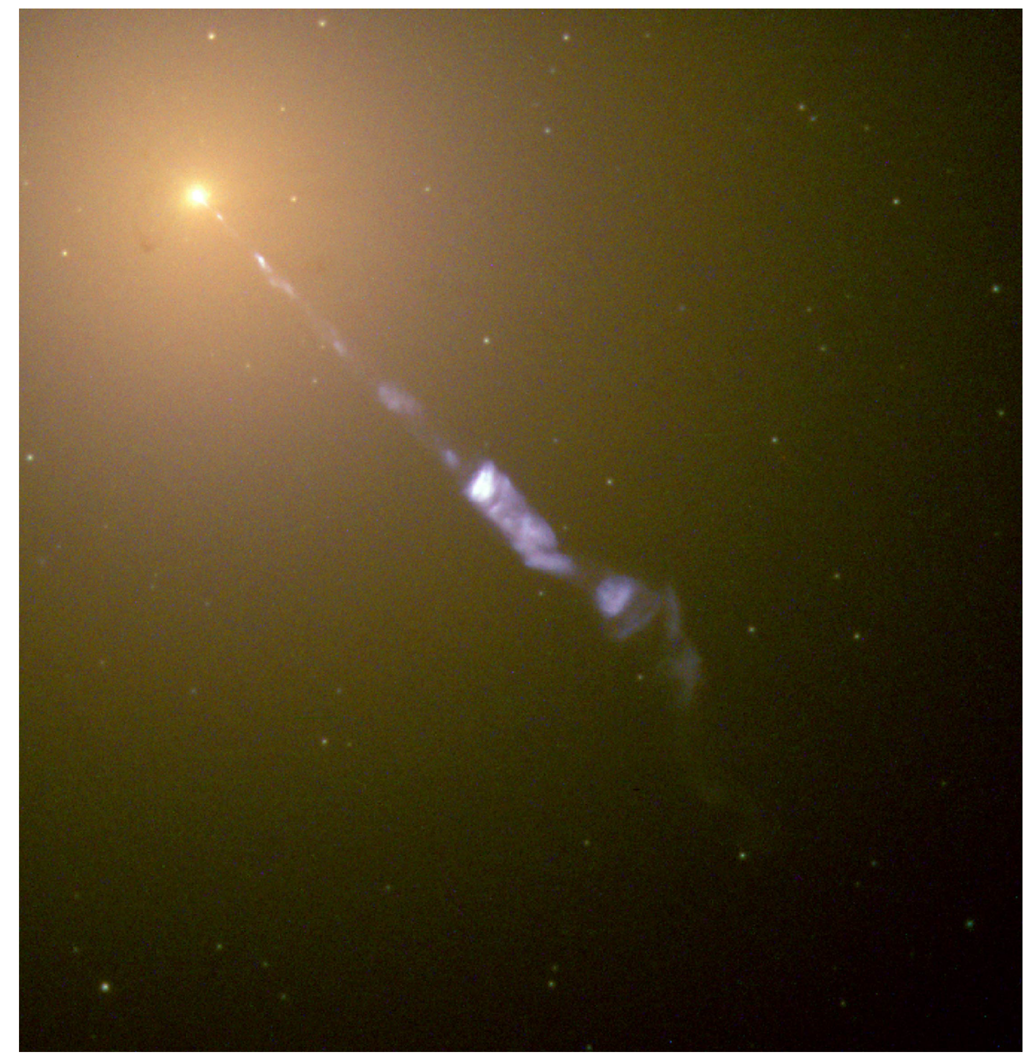


Apparent superluminal motions in ultra-relativistic jets

M87, apparent transverse velocity $v_\perp \approx 6c$

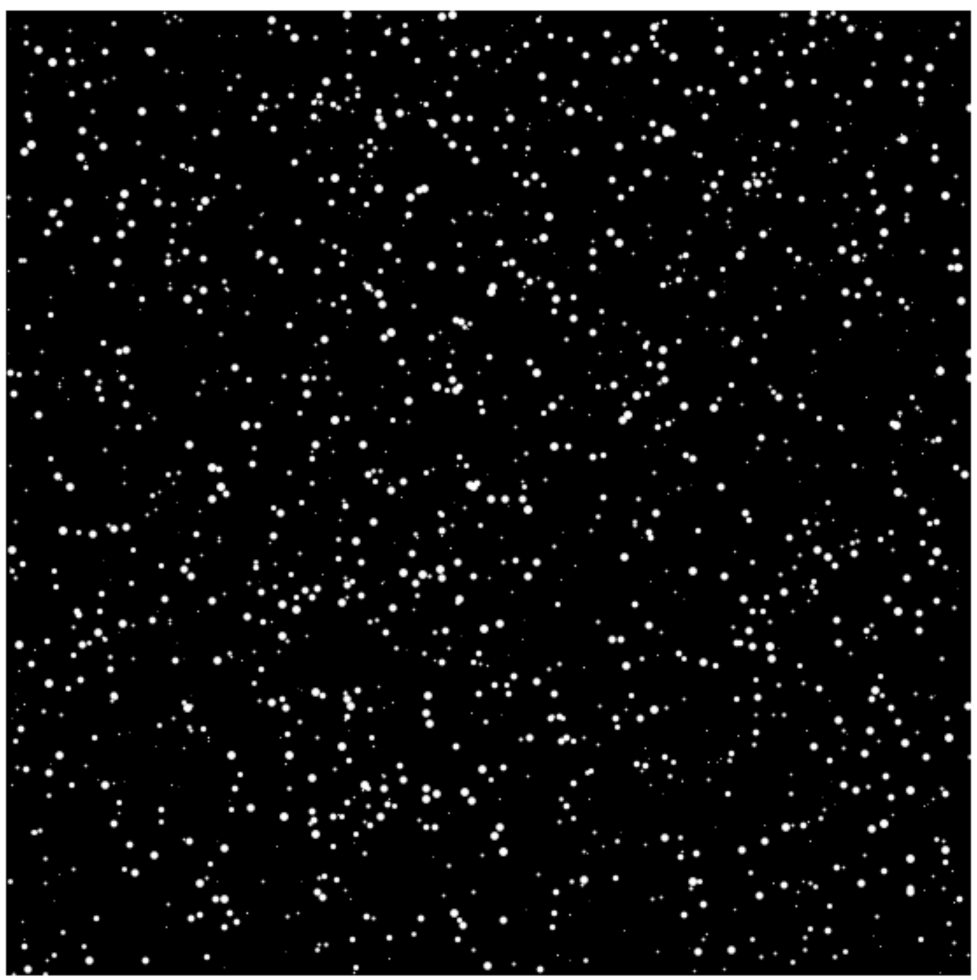


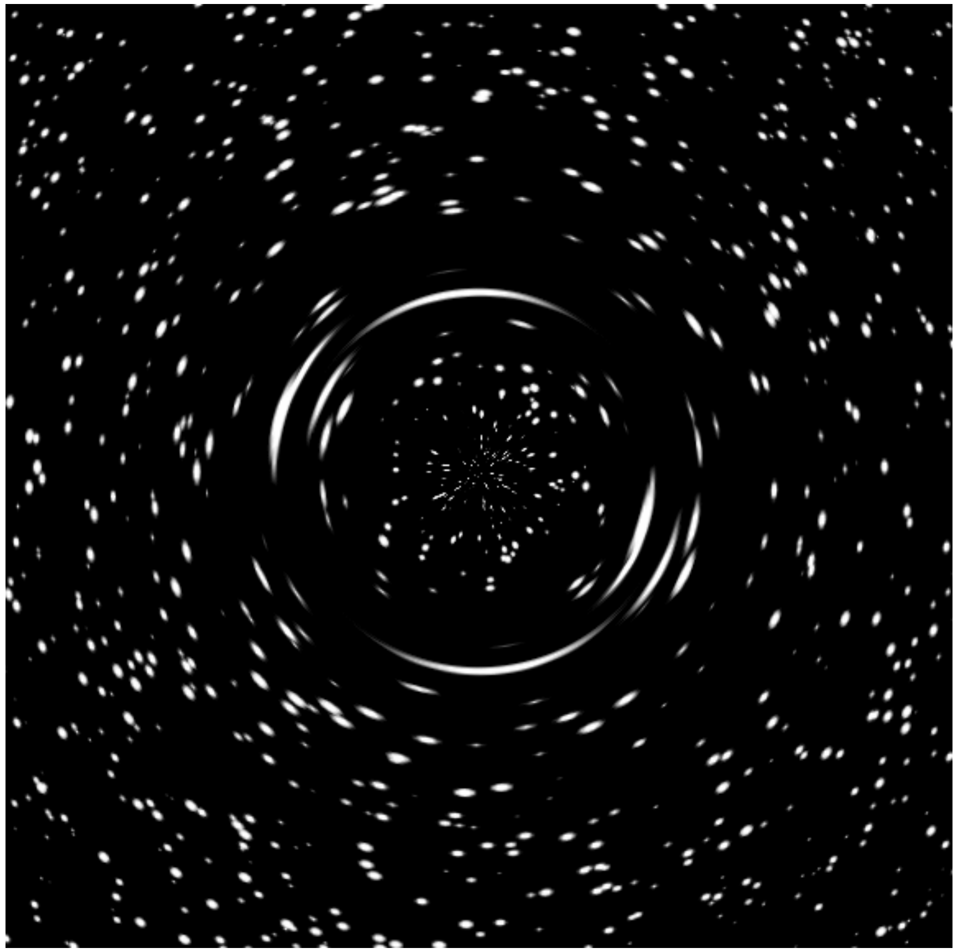
M87 jet
Credits: NASA and the Hubble Heritage Team (STScI/AURA)

Lensing and position drift

$$\delta_{\mathcal{O}} r^A = (u^\mu_{\mathcal{O}}\, l_{\mathcal{O}\,\mu})^{-1}\, {\mathcal{D}^{-1}}^A{}_B \left(\left(\frac{1}{1+z} u_{\mathcal{E}} - \hat{u}_{\mathcal{O}} \right)^B - m^B{}_\nu\, u^\nu_{\mathcal{O}} \right) + w^A_{\mathcal{O}}$$

Lensing by a point source, thin lens approximation

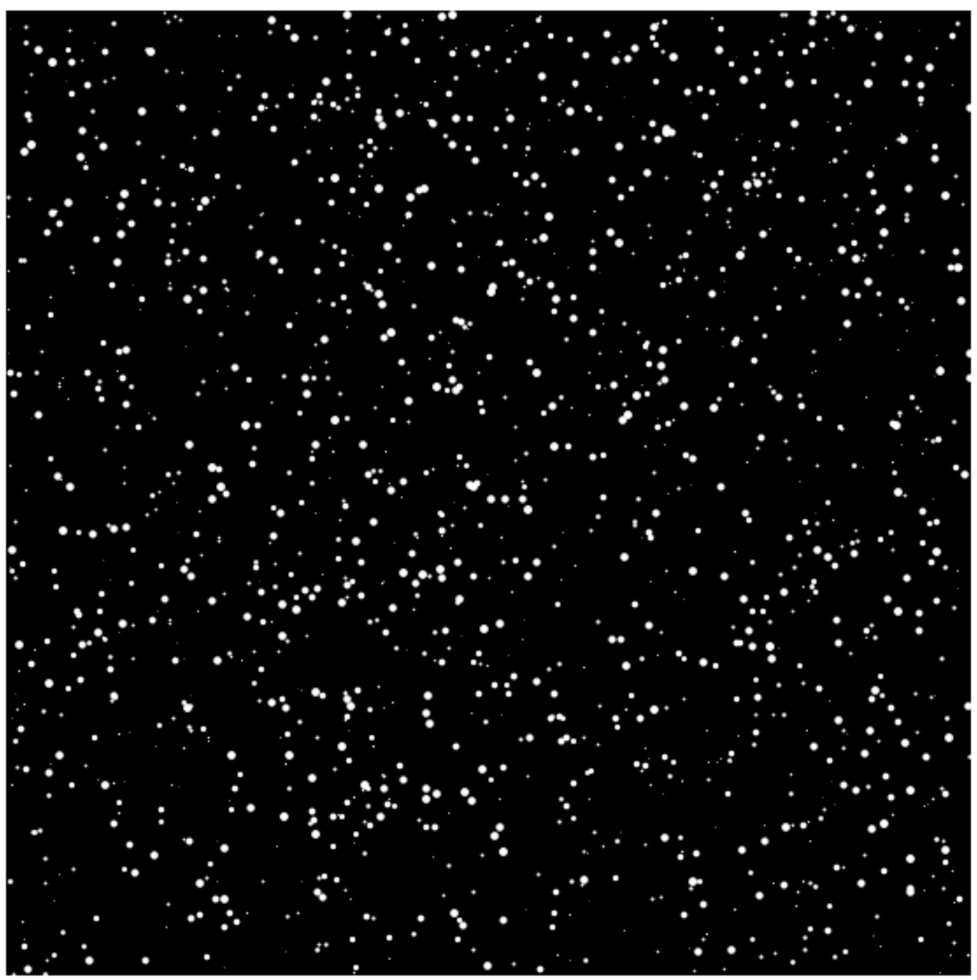


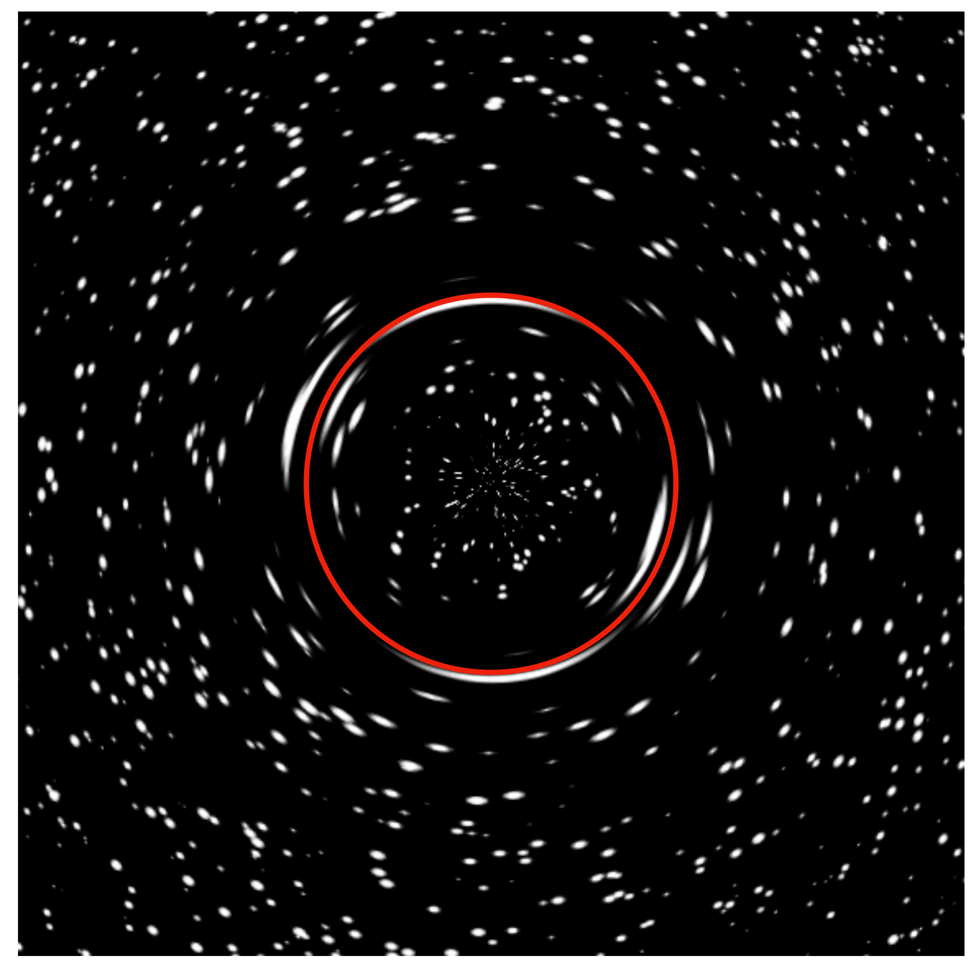


Lensing and position drift

$$\delta_{\mathcal{O}} r^A = (u^\mu_{\mathcal{O}}\, l_{\mathcal{O}\,\mu})^{-1}\, \mathcal{D}^{-1\,A}{}_B \left(\left(\frac{1}{1+z} u_{\mathcal{E}} - \hat{u}_{\mathcal{O}} \right)^B - m^B{}_\nu\, u^\nu_{\mathcal{O}} \right) + w^A_{\mathcal{O}}$$

Lensing by a point source, thin lens approximation

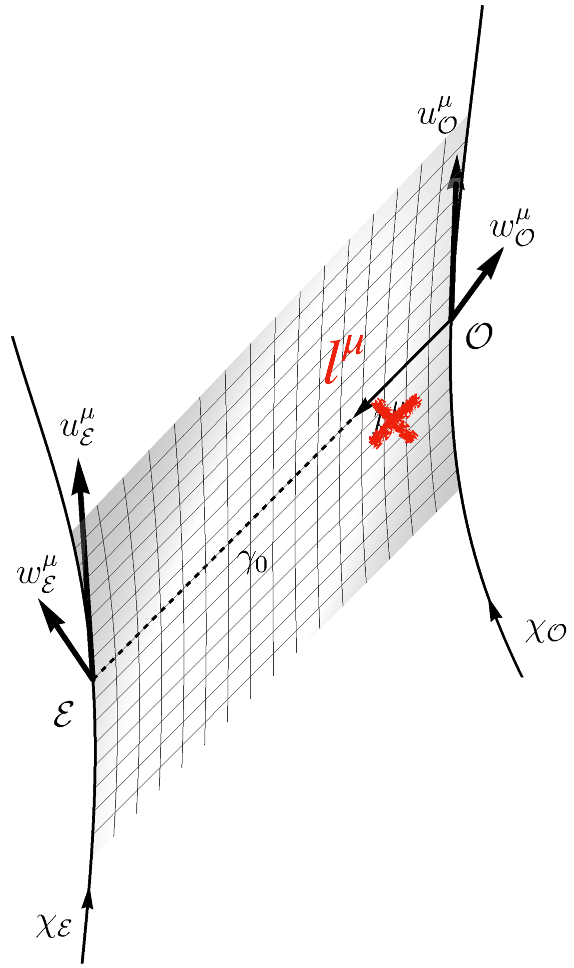




Redshift drift

many expressions possible

$$\ln(1+z) = \ln\left(l_{\mathscr{E}\,\mu}\, u^{\mu}_{\mathscr{E}}\right) - \ln\left(l_{\mathscr{O}\,\mu}\, u^{\mu}_{\mathscr{O}}\right) \quad \Big|\, \nabla_X$$



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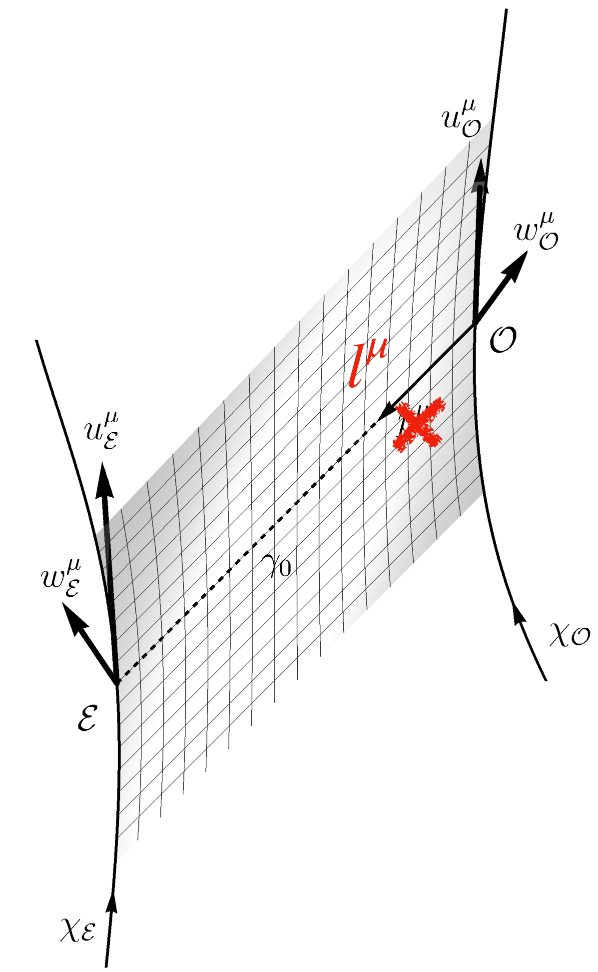
$$\ln(1+z) = \ln\left(l_{\mathscr{E}\,\mu}\, u^{\mu}_{\mathscr{E}}\right) - \ln\left(l_{\mathscr{O}\,\mu}\, u^{\mu}_{\mathscr{O}}\right) \quad \Big| \nabla_X$$

$$\nabla_X \ln(1+z) = \Xi_{Doppler} + \Xi_{Shklovskii} + \frac{1}{l_{\mathscr{E}\,\nu}\, u^{\nu}_{\mathscr{E}}} \int_{\lambda_{\mathscr{O}}}^{\lambda_{\mathscr{E}}} R_{\mu\nu\alpha\beta}\, l^{\mu}\, \hat{u}^{\nu}_{\mathscr{E}}\, l^{\alpha}\, X^{\beta}\, d\lambda$$



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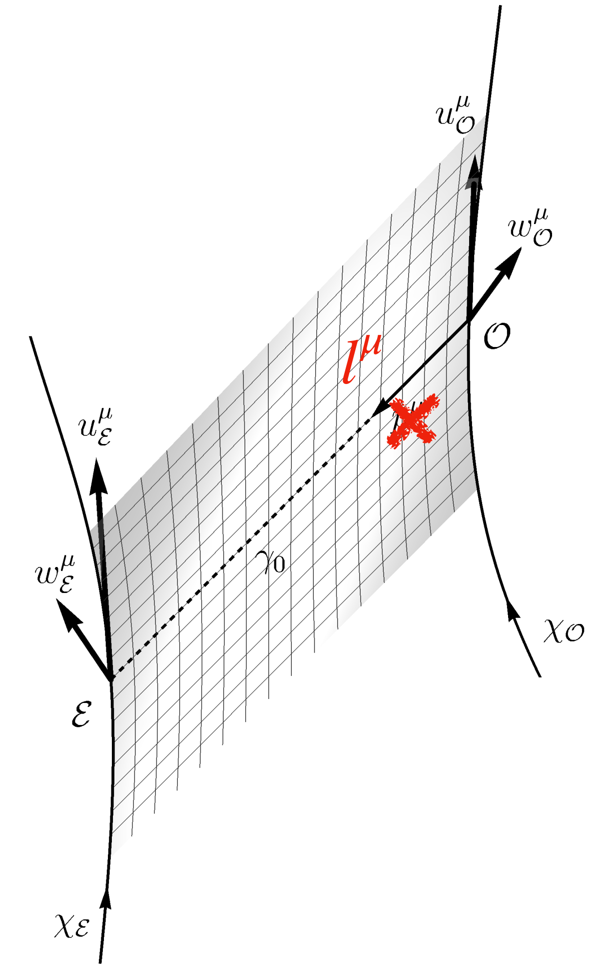

line-of-sight Doppler term (SR)

$$\Xi_{Doppler} = \frac{l_{\mathscr{O}\,\mu}\, w^{\mu}_{\mathscr{O}}}{l_{\mathscr{O}\,\nu}\, u^{\nu}_{\mathscr{O}}} - \frac{1}{1+z}\, \frac{l_{\mathscr{E}\,\mu}\, w_{\mathscr{E}}}{l_{\mathscr{E}\,\nu}\, u^{\nu}_{\mathscr{E}}}$$

Redshift drift

many expressions possible

$$\ln(1+z) = \ln\left(l_{\mathscr{E}\,\mu}\, u^{\mu}_{\mathscr{E}}\right) - \ln\left(l_{\mathscr{O}\,\mu}\, u^{\mu}_{\mathscr{O}}\right) \quad \Big| \, \nabla_X$$

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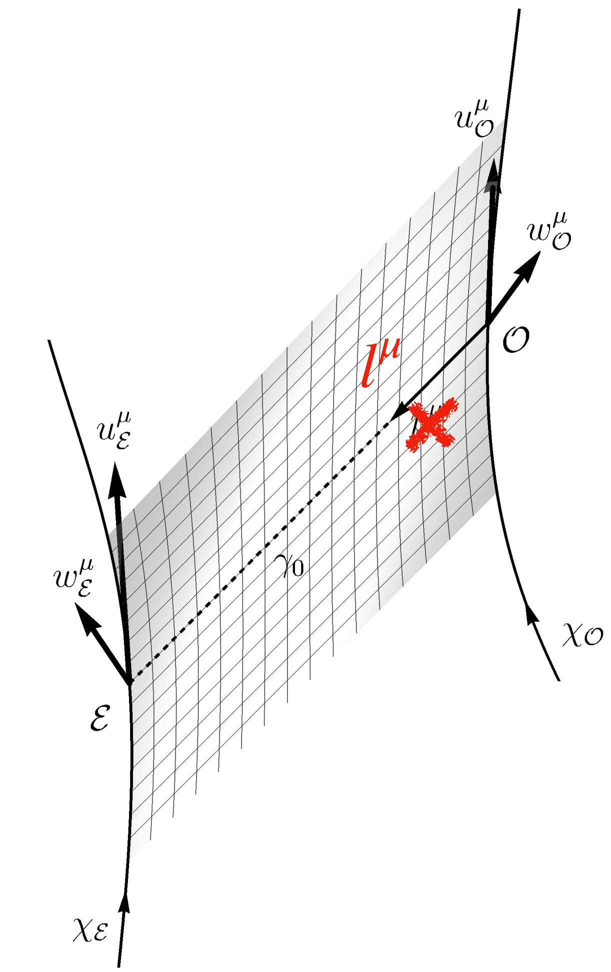

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$$\Xi_{Doppler} = \frac{l_{\mathscr{O}\,\mu}\, w^{\mu}_{\mathscr{O}}}{l_{\mathscr{O}\,\nu}\, u^{\nu}_{\mathscr{O}}} - \frac{1}{1+z}\, \frac{l_{\mathscr{E}\,\mu}\, w_{\mathscr{E}}}{l_{\mathscr{E}\,\nu}\, u^{\nu}_{\mathscr{E}}}$$

Generalized Shklovskii term (SR + GR)

$$\Xi_{Shklovskii} = \frac{\nabla_l X^{A}(\mathscr{O})}{l_{\mathscr{O}\,\nu}\, u^{\nu}_{\mathscr{O}}} \left(\frac{1}{1+z}\hat{u}_{\mathscr{E}} - u_{\mathscr{O}}\right)_A$$

Redshift drift

- line-of-sight Doppler term

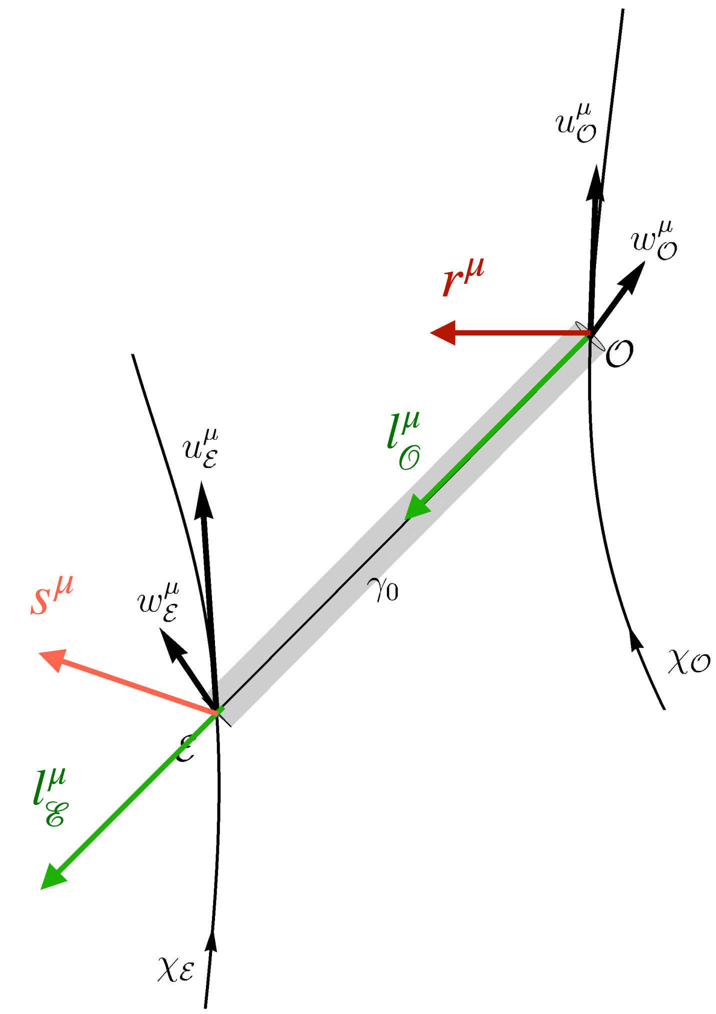
$$\Xi_{Doppler} = \left(\frac{1}{(1+z)^2}\hat{w}_{\mathcal{E}} - w^\mu_{\mathcal{O}}\right)\frac{l_{\mathcal{O}\,\mu}}{l_{\mathcal{O}\,\nu}\,u^\nu_{\mathcal{O}}}$$

$$s^\mu \equiv u^\mu_{\mathcal{E}} + \frac{l^\mu_{\mathcal{E}}}{l_{\mathcal{E}\,\sigma}\,u^\sigma_{\mathcal{E}}}$$

$$\Xi_{Doppler} = \frac{1}{1+z}s_\mu\,w^\mu_{\mathcal{E}} - r_\mu\,w^\mu_{\mathcal{O}}$$

$$s^\mu\,u_{\mathcal{E}\,\mu} = 0$$

$$s^\mu\,s_\mu = 1$$



Variation of relative radial velocity

Variation of the line-of-sight Doppler effect

Pure SR effect, no curvature

Redshift drift

- Generalized Shklovskii term

$$\Xi_{Shklovskii} = \left(\delta_{\mathcal{O}} r^A - w^A_{\mathcal{O}}\right) \left(\frac{1}{1+z}\hat{u}_{\mathcal{E}} - u_{\mathcal{O}}\right)_A$$

$$\Xi_{Shklovskii} = (l_{\mathcal{O}\,\nu}\, u^{\nu}_{\mathcal{O}})^{-1}\, \mathscr{D}^{-1}{}_{AB} \left(\left(\frac{1}{1+z}\hat{u}_{\mathcal{E}} - u_{\mathcal{O}}\right)^A - m^A{}_{\mu}\, u^{\mu}_{\mathcal{O}}\right) \left(\frac{1}{1+z}\hat{u}_{\mathcal{E}} - u_{\mathcal{O}}\right)^B$$

Redshift drift

- Generalized Shklovskii term

$$\Xi_{Shklovskii} = \left(\delta_{\mathcal{O}} r^A - w^A_{\mathcal{O}}\right) \left(\frac{1}{1+z}\hat{u}_{\mathcal{E}} - u_{\mathcal{O}}\right)_A$$

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flat spacetime

$$\Xi_{Shklovskii} = \frac{1}{D_{\mathcal{O}}} \left(\frac{1}{1+z}u_{\mathcal{E}} - u_{\mathcal{O}}\right)_A \left(\frac{1}{1+z}u_{\mathcal{E}} - u_{\mathcal{O}}\right)^A = D_{\mathcal{O}}(\delta_{\mathcal{O}} r^A - w^A_{\mathcal{O}})\,(\delta_{\mathcal{O}} r_A - w_{\mathcal{O}\,A})$$

Redshift drift

- Generalized Shklovskii term

$$\Xi_{Shklovskii} = \left(\delta_{\mathscr{O}} r^A - w^A_{\mathscr{O}}\right) \left(\frac{1}{1+z}\hat{u}_{\mathscr{E}} - u_{\mathscr{O}}\right)_A$$

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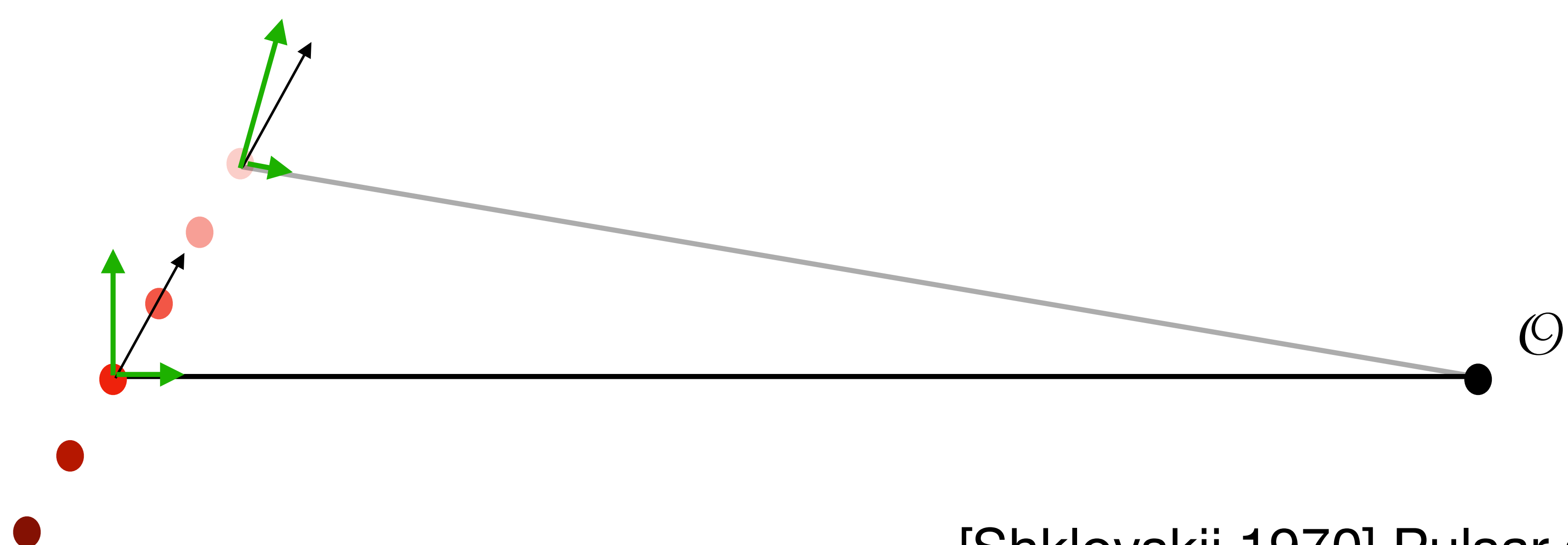

[Shklovskii 1970] Pulsar secular slowdown

Redshift drift

- Generalized Shklovskii term

$$\Xi_{Shklovskii} = \left(\delta_{\mathcal{O}} r^A - w^A_{\mathcal{O}}\right) \left(\frac{1}{1+z}\hat{u}_{\mathcal{E}} - u_{\mathcal{O}}\right)_A$$

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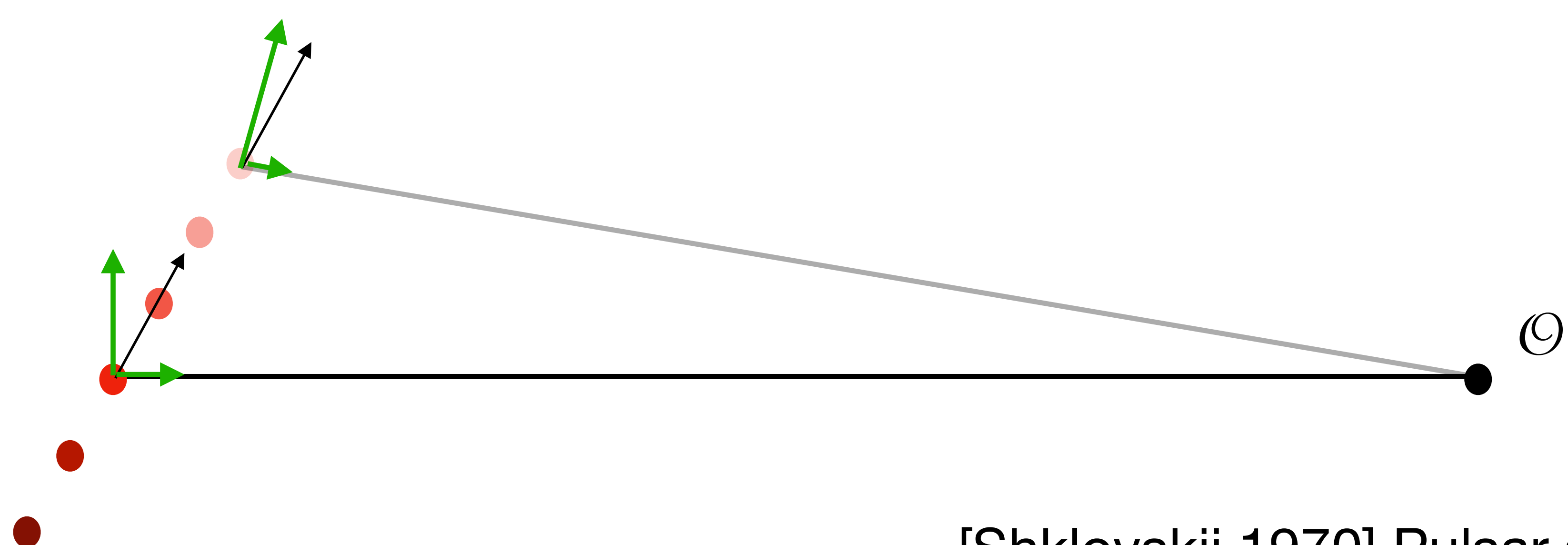


[Shklovskii 1970] Pulsar secular slowdown

SR effect with GR corrections

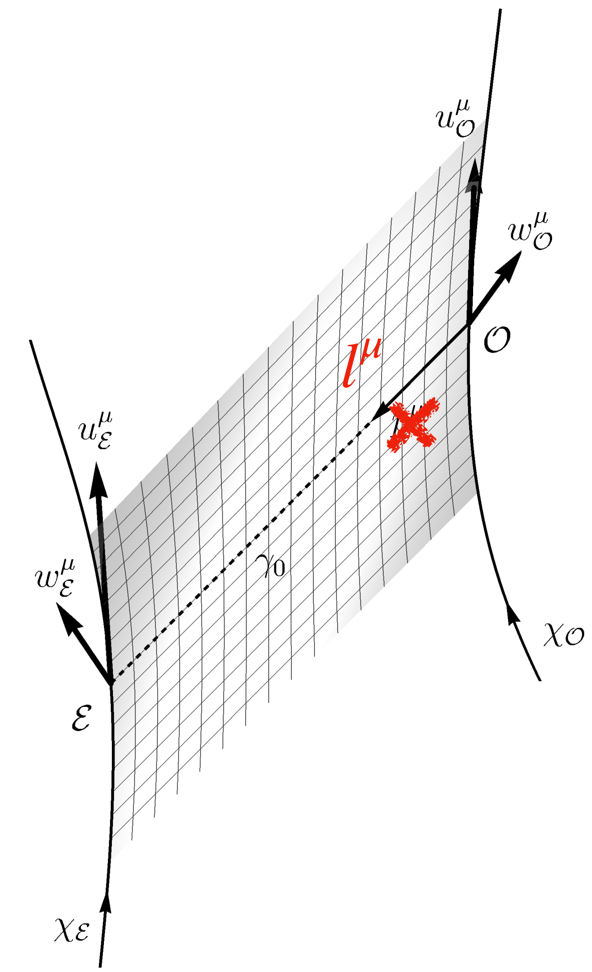
Redshift drift

- Curvature term

$$\frac{1}{l_{\mathscr{E}\,\nu}\, u^{\nu}_{\mathscr{E}}} \int_{\lambda_{\mathscr{O}}}^{\lambda_{\mathscr{E}}} R_{\mu\nu\alpha\beta}\, l^{\mu}\, \hat{u}^{\nu}_{\mathscr{E}}\, l^{\alpha}\, X^{\beta}\, d\lambda$$

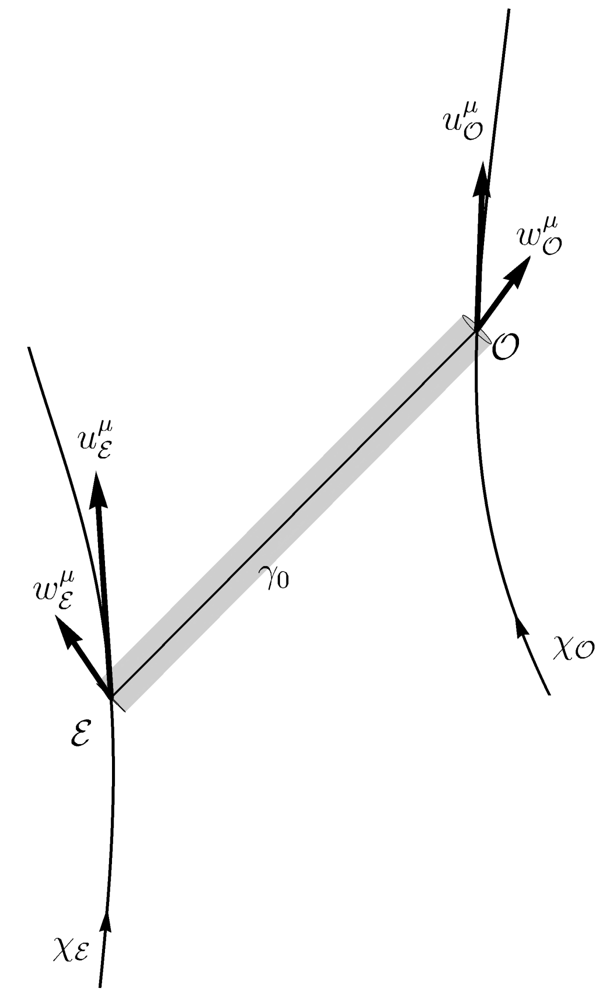
$$X^{\mu} = \hat{u}^{\mu}_{\mathscr{O}} + e^{\mu}_{A}\left(\phi^{A} + m^{A}{}_{\nu}\, u^{\nu}_{\mathscr{O}}\right) + C \cdot l^{\mu}$$

$$\phi^{C}(\lambda) = \mathscr{D}^{C}{}_{A}(\lambda)\, \mathscr{D}^{-1A}{}_{B}(\mathscr{E}) \left(\left(\frac{1}{1+z} u_{\mathscr{E}} - \hat{u}_{\mathscr{O}} \right)^{B} - \tilde{m}^{B} \right)$$



GR/curvature effect

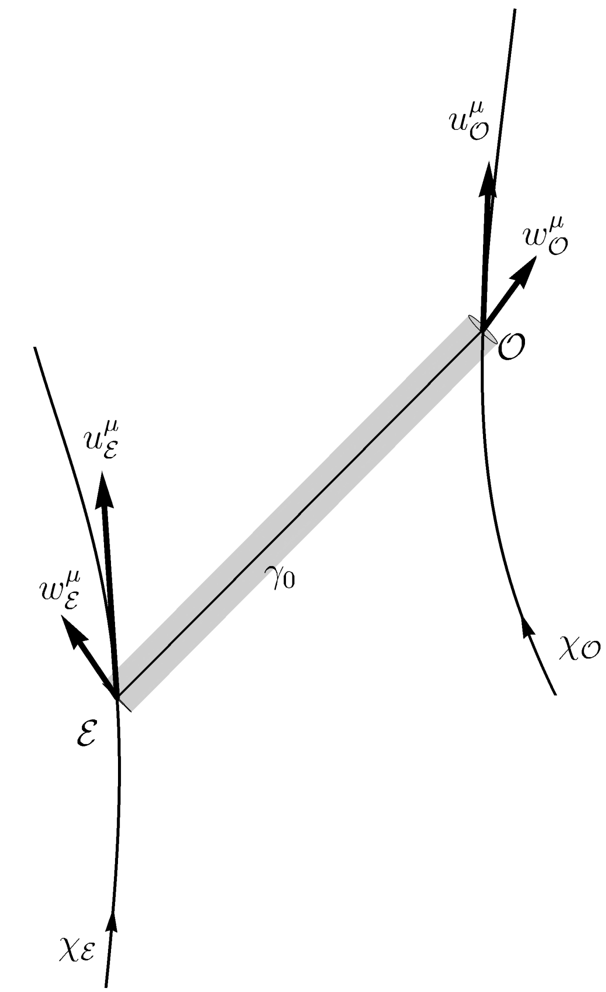
Summary



$u^\mu_\mathcal{O}$
$w^\mu_\mathcal{O}$
$\mathcal{O}$
$u^\mu_\mathcal{E}$
$w^\mu_\mathcal{E}$
γ_0
$\chi_\mathcal{O}$
$\mathcal{E}$
$\chi_\mathcal{E}$

Summary

It is possible to derive exact formulas for the position and redshift drift for any pair of observer/emitter in general relativity (geometric optics approximations)



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Drift rates depend on the spacetime curvature along the LOS and kinematical variables:

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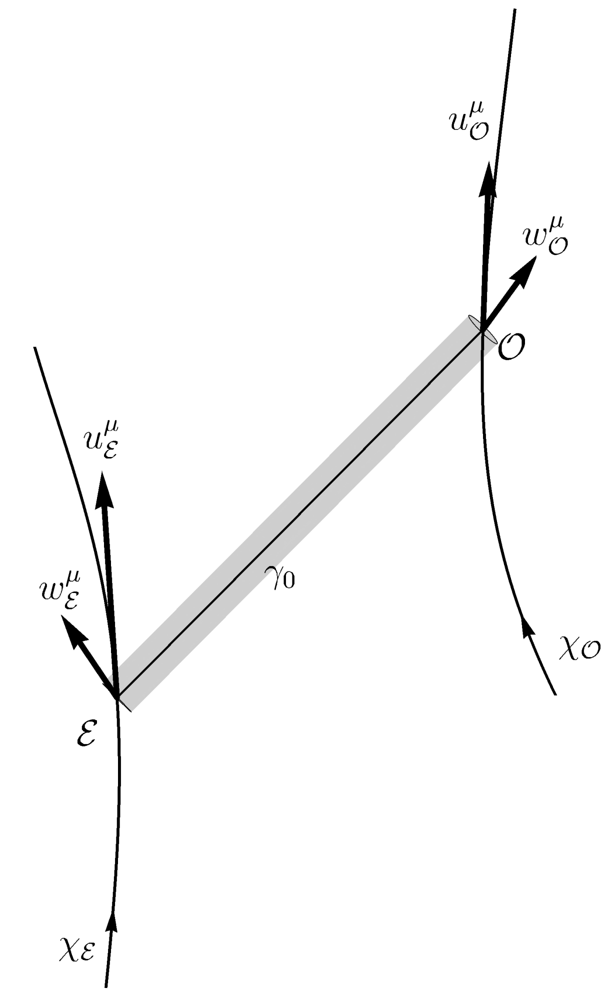

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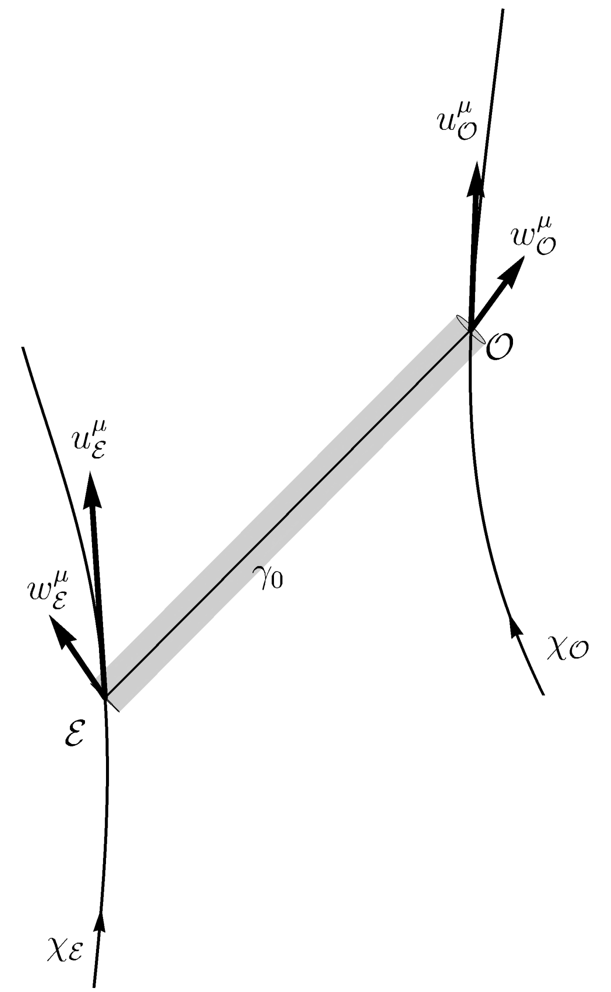
General, non-perturbative relations between:

gravitational lensing and position drift

position drift and the redshift drift



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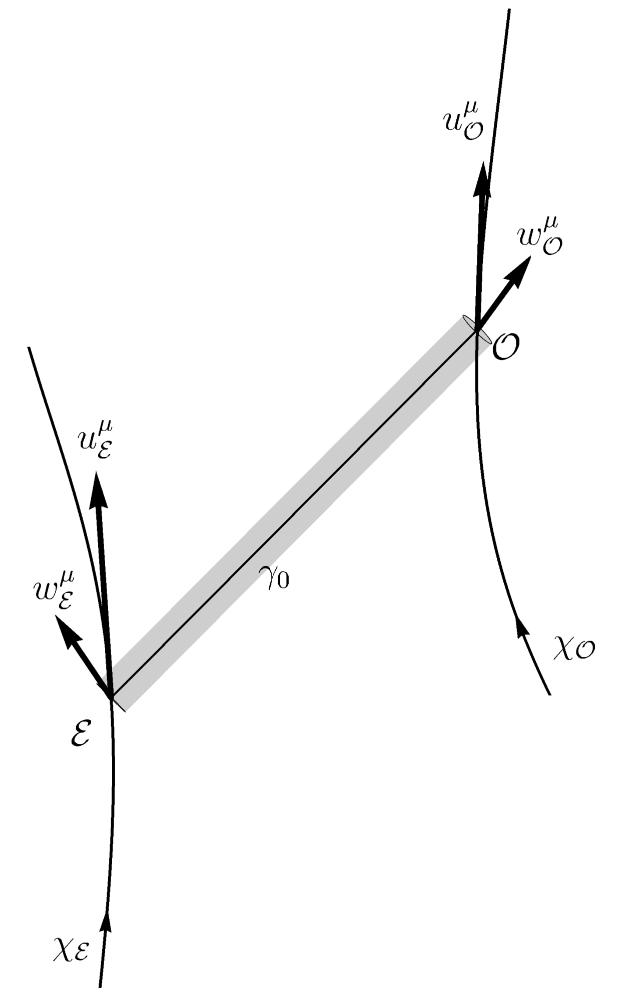

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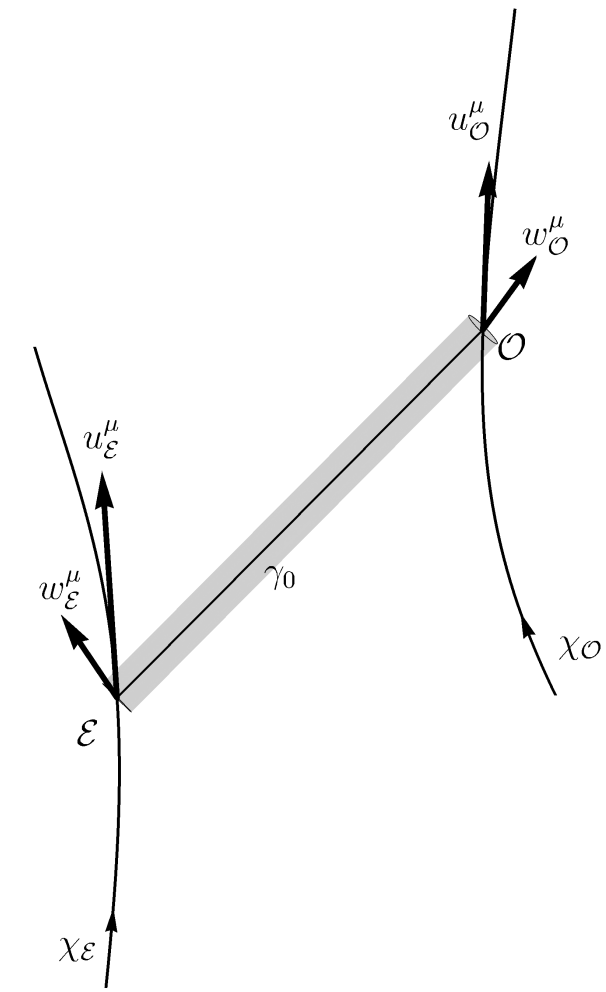

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Thank you!

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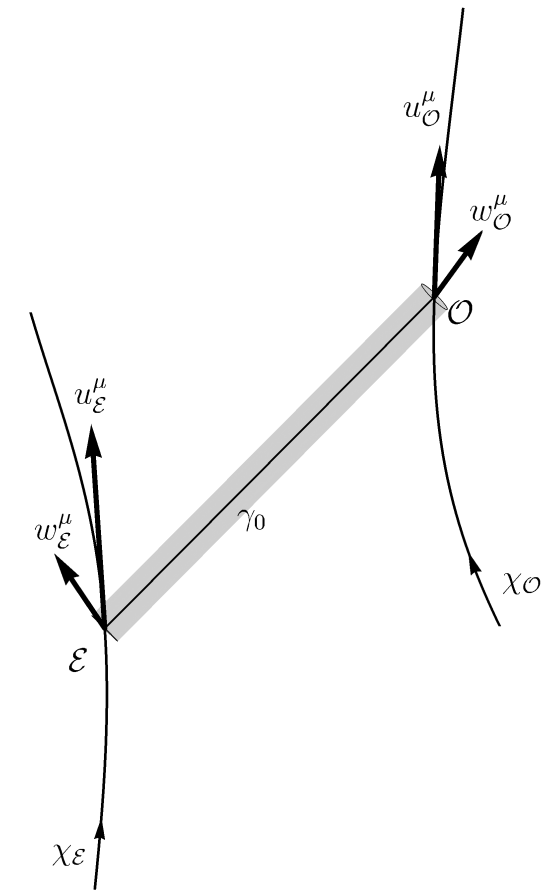
Position and redshift drifts

Exact expressions for drifts - what are they good for?

Sachs formalism for distance measures, image distortions etc.

Limited to momentary observations

Extending the Sachs approach to time variations of observables



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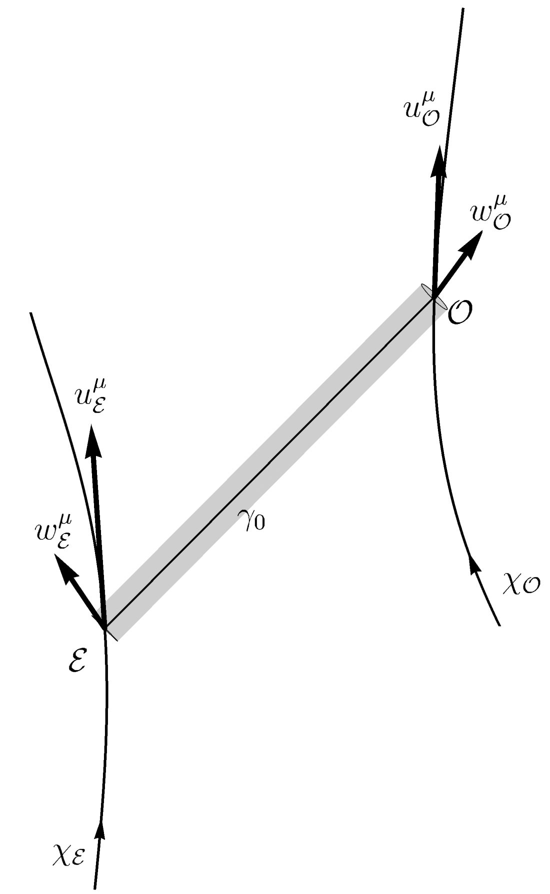

Properties

- Exact - geometric relations. All GR effects automatically involved
- Most general expressions possible - many applications (cosmology, astrometry, pulsar timing…)
- Expressions in terms of kinematical variables (measured wrt local inertial frames at $\mathcal{E}$ and $\mathcal{O}$) and geometry along the line of sight (curvature tensor). Complicated dependence on kinematics separated from the dependence on spacetime geometry
- Starting point for various approximation schemes, useful in numeric [Grasso, Villa, MK 2021]
- Interesting physical consequences (lensing - position drift relation, redshift drift - position drift relation)
- Assumptions: point source, geometric optics, $\mathcal{E}$ not at a caustic
- Main tool: 1st order geodesic deviation equation along a null geodesic